Equals

Realising potential in mathematics for all

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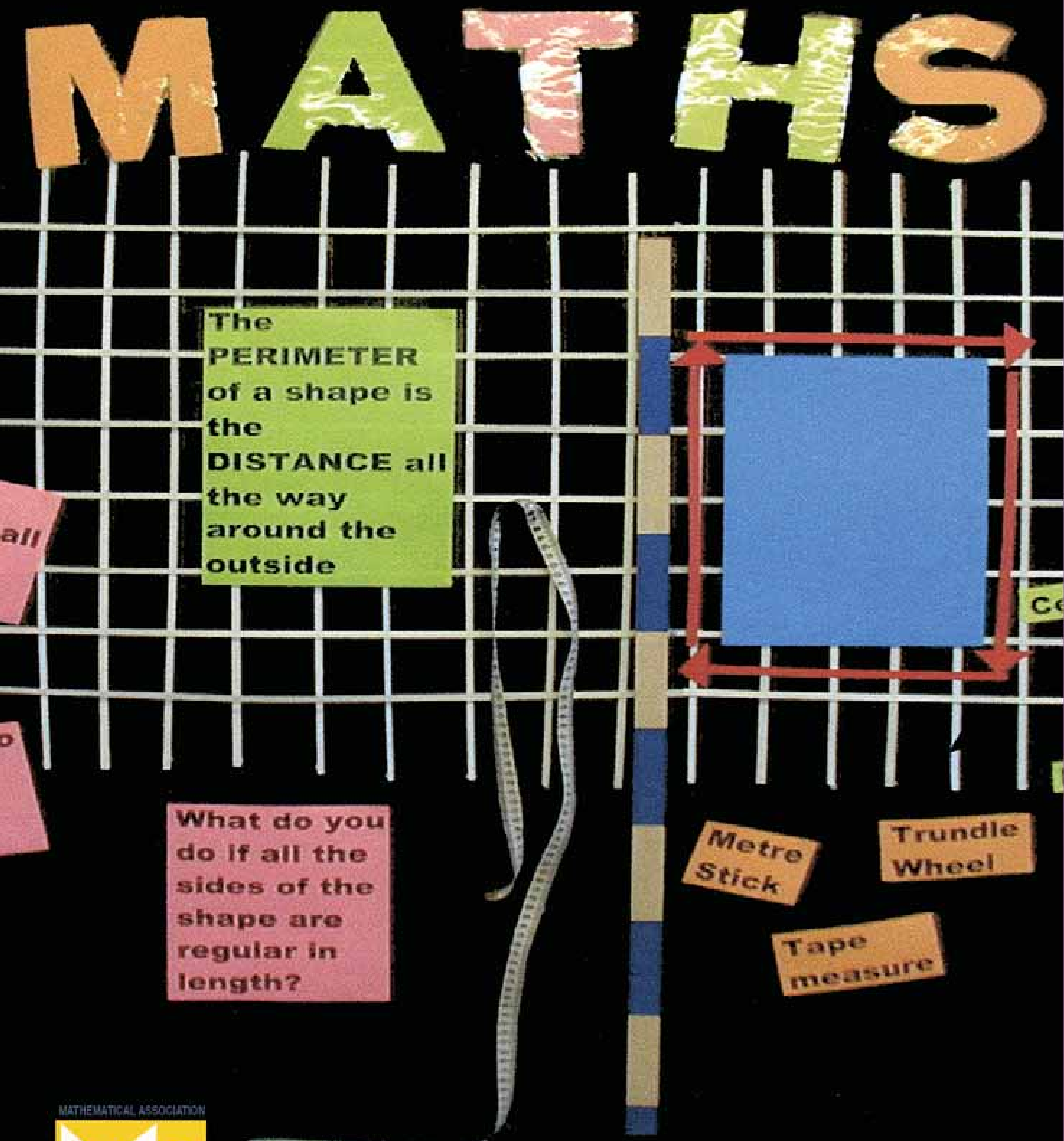

MATHEMATICAL ASSOCIATION
supporting mathematics in education

Realising potential in mathematics for all

Front Cover image from Jennie Pennant

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Editorial

So many numbers!
On February 14th 2006 you will have to use your PIN number to be able to use a credit or debit card. So if you are buying that special present for a loved one for Valentine's Day be warned! Maybe Roy Hudd's pictures will help you (see page 3).

I don't know about you but remembering key numbers and passwords is becoming an increasingly onerous task. We are always warned to destroy the numbers and letters but it is impossible to remember them all. Add to that the length of mobile phone numbers and the myriad amount of other pieces of alphanumeric information we need to remember today and it is not surprising that many people have difficulties. Perhaps these difficulties are not dissimilar from the difficulties that many pupils have in remembering number 'facts'. What you really need if you do forget 'number facts' (and we all do if we are not using them all the time) is an easy way to build them up from scratch. Stewart Fowlie's article on different ways of handling simple number work may be helpful here.

Assessment *for* Learning and appropriate levels
If we are to help children make progress, especially the ones with whom *Equals* is most concerned, - it is not only important to understand how they learn, but also to assess the progress they are making. Without such assessment we cannot plan appropriate learning pathways for them. We are therefore delighted to publish the first part of a series of articles by Dylan Wiliam with his thoughts on assessment *for* learning. In an education system which seems so into assessment *of* learning, Dylan's hypothesis that doing the 'for' will impact on the 'of' is something we could all adopt as a new year's resolution. I was intrigued by the fact that Japanese teachers 'spend a substantial proportion of their lesson preparation time *working together* to devise questions'. The more I hear about the Japanese education system the more I realise that my stereotypical perception of silent classrooms where children learn by rote is probably completely wrong! If you know about mathematics teaching in Japan please let us know! Certainly the results indicate they must be doing something right!!

Continuing the theme of assessment, Nick Peacey discusses the P levels, which have probably been largely ignored by schools other than Special Schools to date. In the nation's drive to achieve targets based on a larger proportion of pupils achieving the average – an interesting thought in itself - the SEN sector has been largely neglected by the National Strategies until recently. Assessment against the P levels might well be something many mainstream schools could explore and would certainly help them to track pupils who have considerable difficulties with learning mathematics.

Appropriate learning materials
The Wave 3 materials produced by the Primary National Strategy do start to address the needs of those pupils well below the average for their age. Patsy Smith and Marjorie Clementson describe how they raised the awareness of the staff in their school and introduced them to the materials.

As a philatelist I am sure I am being continually conned into buying new stamp issues to keep my collection up to date. There are something like 25 different values of Machin definitives (stamps with just the Queen's Head on them) in current circulation. Jane Gabb details some activities, which show quite clearly that you don't need too many stamps to enable you to make all the necessary amounts to post a letter. (You might, however, need a bigger envelope!! (See the centre spread.)

Jennie Pennant begins to explore the considerable potential of digital cameras. They give so many opportunities for both storing information and linking to mathematics in the 'real world' as suggested in Jennie's article. *We are always interested in receiving digital pictures from you together with ways in which you have used them with your pupils. Please use the e-mail address on page 1 to send them to the editorial team.*

Significant figures
Since *Equals* began we have been publishing 'significant figures', snippets of mathematical information gleaned from all types of publications. Often these are quite amusing, sometimes trivial, but many make us sit up and really think. If we can interpret the numbers they give us a clearer picture of the world. We believe as Howard Gardner does that education is

> to do with fashioning certain kinds of individuals … I crave human beings who understand the world, who gain sustenance from such understanding, and who want – ardently, perennially – to alter it for the better …[1]

Further, beside numbers having a fascination in their own right, they are significant in that they help to explain the world:

> "Many centuries ago Plato and Pythagoras had already found in number the clue to the nature of the universe and to the mystery of beauty."…[2]

Rachel Gibbons explores how being able to interpret data is so important in our understanding of the planet on which we all live and how we interrelate with its other inhabitants. We would love to know how you use the significant figures from *Equals* (or any others) with your pupils.

Happy New Year

As we enter 2006 we on the *Equals* editorial team would like to wish all our readers a very Happy New Year.

1. Howard Gardner, *The Disciplined Mind*, London:Penguin Books, 2000

2. Sylvia Ashton-Warner, *Teacher*, New York: Simon & Schuster, 1986, (first published 1963)

Welcome to a swan on two adjacent lakes watching a diving fish

Martin Marsh

Sitting in a doctor's waiting room recently I was intrigued by a magazine article by Roy Hudd[1] in which he described how he overcame the difficulties of remembering apparently random series of digits. He had invented a code for each digit and made up a story related to them. A slightly altered version of the code and associated pictures is shown below.

0 is a pond or lake
1 is a pole stuck in the ground or a mast
2 is a swan
3 is half a snow man
4 is a yacht
5 is a hook
6 is a diving fish
7 is a pole flying a flag
8 is a snow man
9 is a fish swimming upwards

…and you know it really works! It gives a whole new meaning to times tables. 8 x 9 is a flag with a pole next to a swan, 8 x 7 is a hook and a diving fish. Two swans swimming next to a hook is 15^2. Half a snow man with a pole on a yacht with a mast hooking an upright fish watching a swan diving for a fish hooked to the side of a yacht is π to nine decimal places!

We would be really interested in how our readers remember difficult series of numbers!

Slough LEA

1. Yours Magazine, 6th December 2005, page 23

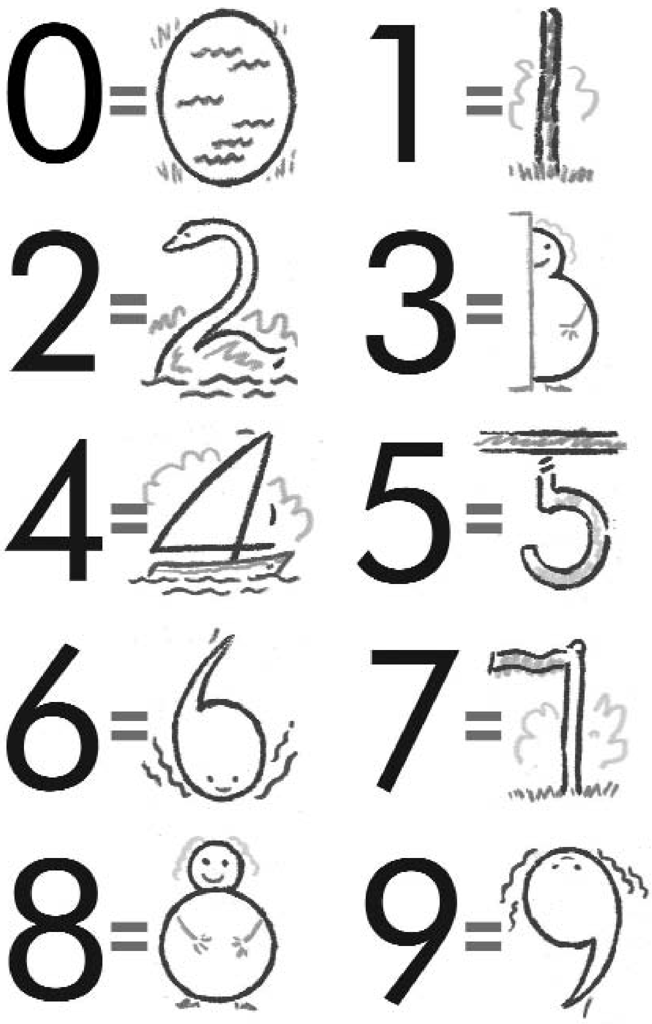


Image Reproduced by kind permission of Stephen May and Yours magazine

Keeping learning on track:

formative assessment and the regulation of learning Part 1

Teaching for deep understanding and raising test scores are not incompatible Dylan Wiliam tells us in this first instalment of his latest paper on assessment for learning. It is interesting to compare these 21st century arguments with those of E. F. O'Neil more than 50 years earlier (see page 20)

Introduction

"I'd love to, teach for deep understanding but I have to raise my students' test scores." I have heard this sentiment from hundreds of teachers in many countries. Implicit in this statement is the notion that raising test scores is not compatible with teaching for deep understanding. As pressures for teachers to be accountable for the performance of their students increase, does this mean that there is no room for teaching for deep understanding? Or is there a way to achieve both?

Over the course of a 10-year study, Paul Black and I sought to find out if using assessment to *support* learning, rather than just to measure its results, can improve students' achievement, even when such achievement is measured in the form of state-mandated tests. In reviewing 250 studies from around the world, published between 1987 and 1998, we found that a focus by teachers on assessment *for* learning, as opposed to assessment *of* learning, produced a substantial increase in students' achievement (Black and Wiliam, 1998a). Since the studies also revealed that day-to-day classroom assessment was relatively rare, we felt that considerable improvements would result from supporting teachers in developing this aspect of their practice (Black and Wiliam, 1998b). The studies did not reveal, however, how this could be achieved and whether such gains would be sustained over an extended period of time.

focus by teachers on assessment *for* learning, as opposed to assessment *of* learning, produced a substantial increase in students' achievement

Since 1999, we have worked with many groups of teachers, from both primary and secondary schools, in the United Kingdom and in the United States. and these collaborations have shown that our initial optimism was justified. In a variety of settings, teachers have found that teaching for deep understanding has resulted in an increase in student performance on externally-set tests and examinations (Wiliam et al, 2004).

The details of how we put these ideas into practice can be found elsewhere (Black et al, 2002; Black et al, 2003). In this paper, I want to describe the key ingredients of formative assessment: effective questioning, feedback, ensuring learners understand the criteria for success, and peer- and self-assessment, and then to show how they fit together within the general idea of the 'regulation of learning'.

What makes a good question?

Two items used in the Third International Mathematics and Science Study (TIMSS) are shown in figure 1 below. Although apparently quite similar, the success rates on the two items were very different. For example, in Israel, 88% of the students answered the first items correctly, while only 46% answered the second correctly, with 39% choosing response (b). The reason for this is that many students, in learning about fractions, develop the naive conception that the largest fraction is the one with the smallest denominator, and the smallest fraction is the one with the largest denominator. This approach leads to the correct answer for the first item, but leads to an incorrect response to the second. Furthermore, if we note that 46% plus 39% is very close to 88%, this provides strong evidence that many students who answered the first item correctly, did so with an incorrect strategy. In this sense, the first item is a much weaker item than the second, because many students can get it.

This illustrates a very general principle in teachers' classroom questioning. By asking questions of students, teachers try to establish whether students have understood what they are meant to be learning, and if students answer the questions correctly, it is tempting to assume that the students' conceptions *match* those of the teacher. However, all that has really been established is that the students' conceptions *fit*, within the limitations of the questions. Unless the questions used are very rich, there will be a number of students who manage to give all the right responses, while having very different conceptions from those intended.

Items that ... provide a "window into thinking"—are difficult to generate

Item1 (success rate 88%)
Which fraction is the smallest?
a) $\frac{1}{6}$ b) $\frac{2}{3}$ c) $\frac{1}{3}$ d) $\frac{1}{2}$

Item 2 (success rate 46%)
Which fraction is the largest?
a) $\frac{4}{5}$ b) $\frac{3}{4}$ c) $\frac{5}{8}$ d) $\frac{7}{10}$

Figure 1 : t wo i tems f rom t he T hird I nternational Mathematics and Science Study

A particularly stark example of this is the following pair of simultaneous equations:

$$3a = 24$$
$$a + b = 16$$

Many students find this difficult, saying that it can't be done. The teacher might conclude that they need some more help with equations of this sort, but the most likely reason for the difficulties with this item is not to do with mathematical skills but with their *beliefs*. If the students are encouraged to talk about their difficulty, they often say things like, "I keep on getting b is 8, but it can't be because a is". The reason that many students have developed such a belief is, of course, that before they were introduced to solving equations, they will probably have been practising substitution of numbers into algebraic formulas, where each letter stood for a different number. Although the students will not have been taught that each letter must stand for a different number, they have generalised implicit rules from their previous experience, just as, because we always show them triangles where the lowest side is horizontal, they talk of "upside-down triangles" (Askew and Wiliam, 1995).

what students learn is not necessarily what the teacher intended

The important point here is that we would not have known about these unintended conceptions if the second equation had been a + b = 17 instead of a + b = 16. Items that reveal unintended conceptions—in other words that provide a "window into thinking"—are difficult to generate, but they are crucially important if we are to improve the quality of students' mathematical learning.

Some people have argued that these unintended conceptions are the result of poor teaching. If only the teacher had phrased their explanation more carefully, had ensured that no unintended features were learnt alongside the intended features, then these misconceptions would not arise.

But this argument fails to acknowledge two important points. The first is that this kind of over-generalisation is a fundamental feature of human thinking. When young children say things like "I spended all my money", they are demonstrating a remarkable feat of generalisation. From the huge messiness of the language that they hear around them, they have learnt that to create the past tense of a verb, one adds "d" or "ed". In the same way, if one asks young children what causes the wind, the most common answer is "trees". They have not been taught this, but have observed that trees are swaying when the wind is blowing and (like many politicians) have inferred a causation from a correlation.

The second point is that, even if we wanted to, we are unable to control the student's environment to the extent necessary for unintended conceptions not to arise. For example, it is well known that many students believe that the result of multiplying 2.3 by 10 is 2.30. It is highly unlikely that they have been taught this. Rather this belief arises as a result of observing regularities in what they see around them. The result of multiplying whole-numbers by 10 is "just to add a zero", so why shouldn't that work for all numbers? The only way to prevent students from acquiring this 'misconception' would be to introduce decimals before one introduces multiplying single-digit numbers by 10, which is clearly absurd. The important point is that we must acknowledge that what students learn is not necessarily what the teacher intended, and it is essential that teachers explore students' thinking before assuming that students have 'understood' something. In this sense assessment is the bridge between teaching and learning.

Questions that give us this "window into thinking" are hard to find, but within any school there will be a good selection of rich questions in use—the trouble is that each teacher will have her or his stock of good questions, but these questions don't get shared within the school, and are certainly not seen as central to good teaching.

In most Anglophone countries, teachers spend the majority of their lesson preparation time in marking books, almost invariably doing so alone. In some other countries, the majority of lesson preparation time is spent planning how new topics can be introduced, which contexts and examples will be used, and so on. This is sometimes done individually or with groups of teachers working together. In Japan, however, teachers spend a substantial proportion of their lesson preparation time working together to devise questions to use in order to find out whether their teaching has been successful, in particular through the process known as 'lesson study' (Fernandez & Makoto, 2004).

Now in thinking up good questions, it is important not to allow the traditional concerns of reliability and validity to determine what makes a good question. For example, many teachers think that the following question, taken from the Chelsea Diagnostic Test for Algebra, is 'unfair':

Simplify (if possible): 2a + 5b

This item is felt to be unfair because students 'know' that in answering test questions, you have to do some work, so it must be possible to simplify this expression, otherwise the teacher wouldn't have asked the question. And I would agree that to use this item in a test or an examination where the goal is to determine a student's achievement would probably not be a good idea. But to find out whether students understand algebra, it is a very good item indeed. If in the context of classroom work, rather than a formal test or exam, a student can be tempted to 'simplify' 2a + 5b then I want to know that, because it means that I haven't managed to develop in the student a real sense of what algebra is about.

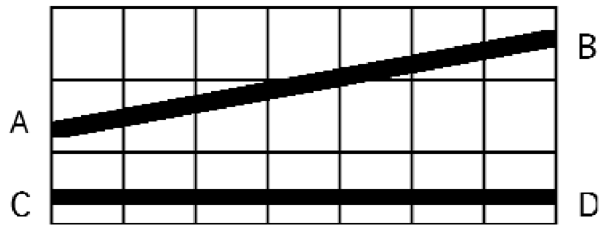
Similar issues are raised by asking students which of the following two fractions is the larger:

$$\frac{3}{7} \qquad \frac{3}{11}$$

Now in some senses this is a 'trick question'. There is no doubt that this is a very hard item, with typically only around one 14-year old in six able to give the correct answer (compared with around three-quarters of 14-year-olds being able to select correctly the larger of two 'ordinary' fractions). It may not, therefore, be a very good item to use in a test of students' achievement. But as a teacher, I think it is very important for me to know if my students think that $\frac{3}{11}$ is larger than $\frac{3}{7}$. The fact that this item is seen as a 'trick question' shows how deeply ingrained into our practice the summative function of assessment is.

A third example, that caused considerable disquiet amongst teachers when it was used in a national test, is based on the following item, again taken from one of the Chelsea Diagnostic Tests:

Which of the following statements is true:
(1) AB is longer than CD
(2) AB is shorter than CD
(3) AB and CD are the same length



Again, viewed in terms of formal tests and examinations, this may be an unfair item, but in terms of a teacher's need to establish secure foundations for future learning, I would argue that this is entirely appropriate.

Rich questions, of the kind described above, provide teachers not just with evidence about what their students can do, but also what the teacher needs to do next, in order to broaden or deepen understanding.

Classroom questioning

There is also a substantial body of evidence about the most effective ways to use classroom questions. In many schools, teachers tend to use questions as a way of directing the attention of the class, and keeping students 'on task', by scattering questions all around the classroom. This probably does keep the majority of students 'on their toes' but makes only a limited contribution to supporting learning. What is far less frequent is to see a teacher, in a whole-class lesson, have an extended exchange with a single student, involving a second, third, fourth or even fifth follow-up question to the student's initial answer. With such questions, the level of classroom dialogue can be built up to quite a sophisticated level, with consequent positive effects on learning.

Of course, changing one's questioning style is very difficult where students are used to a particular set of practices (and may even regard asking supplementary questions as 'unfair'). And it may even be that other students see extended exchanges between the teacher and another student as a chance to relax and go 'off task', but as soon as students understand that the teacher may well be asking them what they have learned from a particular exchange between another student and the teacher, their concentration is likely to be quite high.

students' learning is enhanced by getting them to generate their own questions

How much time a teacher allows a student to respond before evaluating the response is also important. It is well known that teachers do not allow students much time to answer questions, and, if they don't receive a response quickly, they will 'help' the student by providing a clue or weakening the question in some way, or even moving on to another student. However, what is not widely appreciated is that the amount of time between the student providing an answer and the teacher's evaluation of that answer is just as, if not more, important. Of course, where the question is a simple matter of factual recall, then allowing a student time to reflect and expand upon the answer is unlikely to help much. But where the question requires thought, then increasing the time between the end of the student's answer and the teacher's evaluation from the average 'wait-time' of less than a second to three seconds, produces measurable increases in learning (although increases beyond three seconds have little effect, and may cause lessons to lose pace.)

In fact, questions need not always come from the teacher. There is substantial evidence that students' learning is enhanced by getting them to generate their own questions (Foos et al, 1994). If instead of writing an end-of-topic test herself, the teacher asks the students to write a test that tests the work the class has been doing, the teacher can gather useful evidence about what the students think they have been learning, which is often very different from what the teacher thinks the class has been learning. This can be a particularly effective strategy with disaffected older students, who often feel threatened by tests. Asking them to write a test for the topic they have completed, and making clear that the teacher is going to mark the question rather than the answers, can be hugely liberating for many students.

Some researchers have gone even further, and shown that questions can limit classroom discourse, since they tend to demand a simple answer. There is a substantial body of evidence that classroom learning is enhanced considerably by shifting from asking questions to making statements (Dillon, 1988). For example, instead of asking "Are all squares rectangles", which seems to require a 'simple' yes/no answer, the level of classroom discourse (and student learning) is improved considerably by framing the same question as a statement—"All squares are rectangles", and asking students to discuss this in small groups before presenting a reasoned conclusion to the class.

[Dylan Wiliam continues his discussion of Assessment for Learning in future issues of *Equals*]

Learning and Teaching Research Center at the Educational Testing Service, USA.

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Wiliam, D.; Lee, C.; Harrison, C. & Black, P. J. (2004). Teachers developing assessment for learning: impact on student achievement. *Assessment in Education: Principles Policy and Practice*, **11**(1), 49-65.

Dylan Wiliam's article has already been published in Australia:
Wiliam, D. (2005). Keeping learning on track: formative assessment and the regulation of learning. In M. Coupland, J. Anderson & T. Spencer (Eds.), *Making mathematics vital: proceedings of the twentieth biennial conference of the Australian Association of Mathematics Teachers* (pp. 26-40). Adelaide, Australia: Australian Association of Mathematics Teachers.

It is interesting also to note that much of the article was originally published in *Equals* as three pieces:
Wiliam, D. (1999). Formative assessment in mathematics part 1: rich questioning. *Equals: Mathematics and Special Educational Needs*, 5(2), 15-18.

Wiliam, D. (1999). Formative assessment in mathematics part 2: feedback. *Equals: Mathematics and Special Educational Needs*, 5(3), 8-11.

Wiliam, D. (2000). Formative assessment in mathematics part 3: the learner's role. *Equals: Mathematics and Special Educational Needs*, 6(1), 19-22.

Getting to grips with the P Scales: more than halfway there?

The P Scales, **Nick Peacey** reminds us help to provide a quality of opportunity for all.

A sixteen-level curriculum

'Government needs to find a way of recognising and celebrating the achievements of these pupils and their teachers, often against considerable odds.' (Audit Commission 2004)

The National Curriculum established performance criteria for the assessment of attainment of most school pupils in 1988. Those who did not attain level 1 of the National Curriculum were then recorded as W or 'working towards' [level 1]. This vagueness provoked widespread concern: pressure eventually led to the introduction of the P scales: 'differentiated performance criteria. They outline attainment for pupils working below level 1 in the national curriculum'. (QCA 2005).

The P scales are part of the national curriculum assessment system and are therefore designed for pupils aged 5-16. They are essentially a 'best-fit' teacher assessment system for end of year or end of Key Stage judgement about attainment.

Their creation completed what amounts to a sixteen level curriculum, with P1-P8 running up to (mainstream) levels 1-8. Unfortunately this system has never been published as an entity. The current position for mathematics is that the National Curriculum levels are at www.nc.uk.net and the P levels are in the 3-14/Inclusion section of www.qca.org.uk. Perhaps the next revision of the National Curriculum will bring all of them together.

'Government needs to find a way of recognising and celebrating the achievements of these pupils and their teachers'

Slow progress in the P Levels

All P Scales follow the same model: it is assumed that 'subject learning' starts at P4. So Ps1-3 are generic, the same for every subject; Ps 4-8 are subject-specific scales. We are of course talking about children and young people working at very early levels of development here: for example, at P4 of the Listening/Receptive Communication scale the child is expected to understand only 50 words.

The mathematics scale, like the English and unlike the science one, follows three strands from P4 onwards. These are the same strands as appear in the national curriculum levels: *Using and Applying Mathematics, Number and Shape and Space*. Because we know a fair amount about the early learning of mathematics, the P scales follow a reasonably 'evidence-based' developmental pattern. We might not feel quite so secure about P scales for the less-researched area of ICT learning, to cite one example.

One feature of the scales soon became clear: 'Pupils working at the very lowest P scales …may struggle to move up even one P level during a whole key stage ' (DfES 2004). Teachers of pupils with profound and multiple learning difficulties in this group frequently expressed their worries about the lack of steps within the scales for such pupils. 'Commercial' schemes like *Equals* (this magazine's namesake: a university/schools consortium), *B Squared* (a small company) and *PIVATs* (a Lancashire LEA initiative) began to offer smaller steps and finer divisions of scales.

Assessment at P levels

Currently teachers can use any tool to assess students working on the P levels, including the 'commercial' schemes. This freedom is associated with the assumption that over time the P scales will be adopted throughout the system, in both mainstream and special schools. Unfortunately, government's commitment to this desirable conclusion never took strategic-let alone funded- form. Knowledge of the P scales is therefore patchy. Some local authorities, like Hampshire and Newham, have taken the P scales or their equivalent forward across all their schools; elsewhere, many mainstream schools ignored them, while special schools did their own thing.

The Audit Commission noted this in 2003 when it turned its spotlight on special educational needs.

The Commission found it hard to check value for money spent in relation to pupils on the P scales, who probably benefit from a large proportion of the £3.5 billion spent on SEN across England each year.

'In contrast to the national focus on standards of attainment, little is known about the outcomes achieved by children with SEN. A lack of monitoring of their achievement and a lack of relevant performance measures make it difficult to recognise the good work in many schools, or to identify where children are poorly served….Schools feel pulled in opposite directions by pressures to achieve ever-better academic results and to become more inclusive.'

The P scales are part of the national curriculum assessment system

> 'National performance tables and targets fail to reflect the achievement of many children with SEN…. With resources for SEN increasingly being delegated to school level, it is critical that appropriate accountability structures are in place so that parents can be confident their child's needs are being met. (Audit Commission 2004).
>
> The Government's response in its SEN strategy (DfES 2004) flagged the need for all schools to use the P scales in the cause of accountability and outlined an approach to make them both more user-friendly and more used across the system.
>
> Encouragingly the Strategy gave its support to teacher assessment in all forms. 'We will develop the use of teacher assessment to monitor the progress made by pupils…We will encourage schools to make better use of routine tests, tasks and other forms of assessment to inform target-setting.' (DfES 2004a).

The Mathematics scale from P4 onwards follows the same three strands as the national curriculum levels

Moderation at P levels

Politically, the scope of P scales use is an issue. They were originally devised for target-setting, review and reporting and have been found helpful for that process. They also support curriculum development. Teachers seeking to include a young person with learning difficulties in their lessons have often not spent long in training on any sort of developmental model of how we first learn mathematics (i.e the typical order in which early learning of the subject occurs). The scales offer exactly that. The realisation that a useable map exists can boost confidence for many preparing work for a class. Those whose responsibilities include professional development can acquire the QCA guidance DVD (QCA 2005) and work with colleagues on 'removing barriers' to the curriculum for pupils like Gary and Sophie who are profiled in film on the disc.

For any developed use of P scales a process of moderation will be necessary. This would be expected in relation to GCSE coursework assessment, but not always realised as important in this context. After all, a 'best-fit' level judgement of one teacher may not be the same as that of another. It is clear that no inter-school collaboration in this area can take place without some such process in place. The QCA DVD illustrates moderation well and makes the point that the process can in its own right be valuable professional development.

So schools can collaborate in self-evaluation and monitoring. This is sound and uncontroversial. Some local authorities already gather P scale results and offer schools print-outs of them collated for internal analysis; some schools use systems such as those have been provided by national collation of results or commercial schemes.

Too uncertain for publication?

From time to time over the last few years the government has flirted with taking a step beyond this: the inclusion in some way of P scale results in performance tables. There was even a consultation on the subject. 'We would need to devise a system that ensured schools get at least as much credit for pupils making progress in P scales as they get for other pupils working towards National Curriculum levels' (DfES 2004b). This is an understandable ambition, but not one the P scales were ever designed to fulfil. Regardless of whether they are published or not, there are too many uncertainties about the judgements involved for externally-made school or local authority comparisons to have any validity. So the idea should be treated with caution: if schools can be given credit for rapid pupil progress on the P scales, others can presumably be discredited on what would be assumed to be slow progress.

The government is collecting P scale results from schools at present, but this is on a voluntary basis (as is the use of the scales).

The DfES has invited 'schools to report the attainment of any child with special educational needs working below national curriculum level 1 as a 'P level'.

So public comparisons on the basis of P scales results look unlikely at present. If the idea re-emerges it should be strongly resisted. For now we can bend our energies to encouraging all colleagues working in the mainstream with students at below level 1 of the national curriculum to make full use of the resource. The Disability Discrimination Act 2005 gives all public authorities the duty of promoting disability equality. Pupils working below National Curriculum levels have the same right to proper assessment of their attainment as their peers. All teachers need to understand the possibilities of the P scales to help them avoid discrimination and make 'reasonable adjustments' to the programmes they plan for their students.

SENJIT, Institute of Education, London

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DfES (2004a) *Removing Barriers to Achievement: The Government's [ten year] SEN strategy*
DfES (2004b)*The Consultation on Performance T ables and Pupils with SEN*
QCA (2005) U*sing the P scales: assessing, mode rating and reporting pupil attainment in English, Mathematics and Science at levels P4-P8*
(reference QCA/05/1589. The pack includes a guidance booklet and DVD:it is available without charge from orderline@qca.org.uk)

There's not just mathematics, there's morality

What number knowledge do citizens need to understand the world they live in? asks Rachel Gibbons

The other day, in an entirely non-educational context, I met a woman from Wales who turned out to be a learning support assistant. Because, clearly, we both had an interest in education she told me about a recent set of experiences she had had in school. In one class they had been 'doing mean, median and mode'. In her own school days she herself had been failed by the system, bunked off and generally lost out in mathematics. For this lesson she asked the teacher if she could see the textbook because she knew her own shortcomings and took her work seriously. She wanted to make quite sure she understood enough to guide the children in her care in the right direction. I didn't get all the details of the problem they were dealing with but enough to know that she thought the teacher had made a mistake. She went off to the next class who were on the same topic and again the teacher set a problem to which, this time, she got the right answer. She discussed the problem with the teacher and told her of what she had seen in the previous lesson, saying that the last teacher had done it in a different way. "Well, he was wrong," said teacher number two. Eventually it was discovered that teacher number two, although correct in her theory, was also wrong. She had made a mistake in her calculations.

I think this story is important because it illustrates the state of understanding of mathematics – or lack of understanding – that exists today among teachers and learning support assistants (in Wales if nowhere else). Admittedly, it is only one example but I fear it may be typical. The topic being tackled was statistics, perhaps, in its simplest form, the most frequently applied branch of mathematics in everyday life. You can't get away from them, they are what are used in every newspaper, most adverts and many TV and radio news programmes to explain the characteristics of the world in which we live. What are 'the basics' you need to know to understand statistics?

First, I suggest that, to understand and use statistics, you do need to develop a sense of proportion. In every issue of *Equals* we put some "significant figures" many of which are presented in the form of fractions, percentages or "x in every y …" statements. Let's look back over a selection. What does it mean if you say that:

95% of those killed in wars are civilians - not soldiers

Half the single mothers in Britain according to a recent study have no educational qualifications at all...

...on **average** women still earn **20%** less than men...

Women do $^2/_3$ of the world's work for less than **10%** of the world's salary and own less than **1%** of the world's wealth.

Twenty-eight pence of every pound spent on national lottery tickets is diverted from the coffers of Camelot into "good causes".

In 1999 more than **one in four** 15-year-olds smoke .

10% of all your taxes, including VAT, go to the military.

90% more is spent on promoting arms sales than civil exports.

62% of young people asked admitted to hearing problems after dancing the night away to sounds at 120 decibels.

The richest **fifth** of the world's population now has an income **74 times** that of the poorest **fifth**.

The average school could save **£17.85 per pupil per year** by introducing energy efficiency levels

Only one of those statements mentions the word average but they are all about proportion: percentages, fractions and 'x in every y' statements. What pictures do they create in the mind? What do they mean? Where did they come from? (*Equals*, of course, always quotes a source) Are they reliable? And, bringing in the moral - are the situations described fair?

Dickens, in *Hard Times*, has Sissy Jupe being well aware of the moral dimension when asked the question

> … in a given time a hundred thousand persons went to sea on long voyages, and only five hundred of them were drowned or burnt to death. What is the percentage?

Sissy's reply was, " Nothing, … I said it was nothing.... - to the relations and friends of the people who were killed. I shall never learn."[1]

Can we consider the mathematical meanings of such statements without considering at the same time, like Sissy, the moral meanings?

Maybe this is the area of mathematics we should be tackling in all CPD. With my own phobia for averages, I would suggest that for the 'average' citizen averages are probably the least important ideas in this field. Certainly the exercise of finding the mean, median and mode of a set of figures is completely meaningless and gets one nowhere. I suppose it could be seen as a useful experience in simple arithmetic but, if used for this purpose, doesn't it confirm the view that mathematics is a load of theoretical gobbly-de-gook?

Even the simplest fraction can be viewed and interpreted in such different ways – for example consider the emotive difference between the effects of saying something is half empty and saying that it is half full. Besides statistics needing to engage the pupils, the subject needs to be felt as relevant. Does it really matter which are the favourite colours of the pupils in a class? If the classroom has to be redecorated and the pupils are being allowed to have a say in the colour scheme it might make sense, but otherwise who cares? The proportions of pupils with different coloured eyes could be important if it is linked with some biological theories. But statistics is essentially a branch of applied mathematics and can seem exceedingly dry bones unless some wider curriculum flesh is put on it. The Mathematical Association's response to the original National Curriculum contained these words:

> Much current work on data handling concentrates too much on graphical representation and calculation of averages. These have an important place, but it is encouraging to see greater emphasis on defining problems, using a wide range of data sources and particularly the *interpretation* of what has been found.[2]

Fulham, London

1. Charles Dickens, *Hard Times*. (1854) Penguin edition 1969 p. 97. 98.
2. My italics.

N.B. If these ideas have interested you why not join the *Equals* workshop at the Mathematical Association Easter conference:
It's your Responsibility.
Those who appreciate mathematics should be sailing with the neglected minority – the non-A to C group. What will interest them? What do they need? How do we help those non-mathematicians who do teach them? A workshop on preparing appropriate materials for this group at all ages.

Problem solving with coins and stamps

1. Best coins

You can use one of each of these coins: 1p, 2p, 5p, 10p, 20p, 50p, £1

What amounts can you make (and not make) up to £1?

Use the 100 square to colour in those which you can make.

What patterns can you see in the amounts that can't be made?

Can you explain why it is impossible to make those amounts?

What coins would you need to add in order to make all these amounts?

Extension: If you could make up your own coins, what would you choose to make all the amounts up to £1 using the least number of different coins?

1p	**2p**	**3p**	**4p**	**5p**	**6p**	**7p**	**8p**	**9p**	**10p**
11p	**12p**	**13p**	**14p**	**15p**	**16p**	**17p**	**18p**	**19p**	**20p**
21p	**22p**	**23p**	**24p**	**25p**	**26p**	**27p**	**28p**	**29p**	**30p**
31p	**32p**	**33p**	**34p**	**35p**	**36p**	**37p**	**38p**	**39p**	**40p**
41p	**42p**	**43p**	**44p**	**45p**	**46p**	**47p**	**48p**	**49p**	**50p**
51p	**52p**	**53p**	**54p**	**55p**	**56p**	**57p**	**58p**	**59p**	**60p**
61p	**62p**	**63p**	**64p**	**65p**	**66p**	**67p**	**68p**	**69p**	**70p**
71p	**72p**	**73p**	**74p**	**75p**	**76p**	**77p**	**78p**	**79p**	**80p**
81p	**82p**	**83p**	**84p**	**85p**	**86p**	**87p**	**88p**	**89p**	**90p**
91p	**92p**	**93p**	**94p**	**95p**	**96p**	**97p**	**98p**	**99p**	**100p**

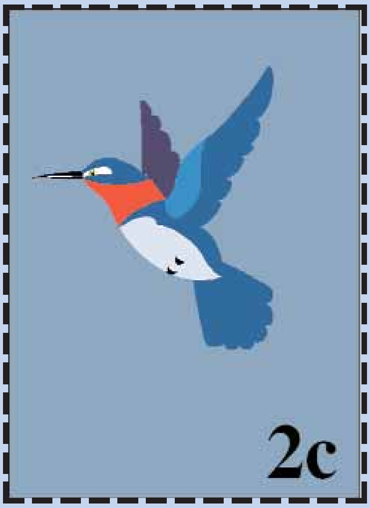
2. Giving change

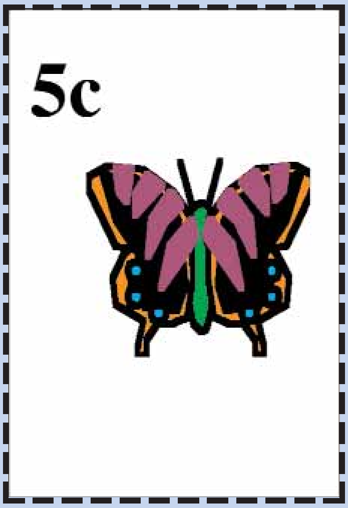
You can use any coins for this activity.

If you had to give someone 5p change, how many different ways could you do it?

What about other amounts of change?

Can you find any patterns in the number of ways of giving change?





3. Strange stamps

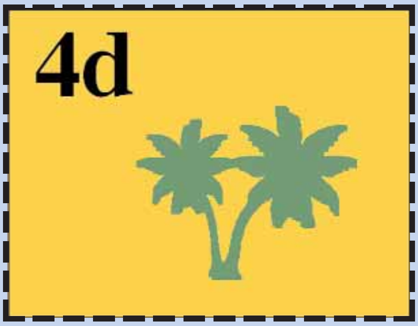
On a small island the only stamps available are 2c and 5c stamps.

What amounts of postage up to 20c can you make with these?

What can't you make?

What is the largest amount that you can't make?

On another island the stamps available are 3d and 4d.



What amounts of postage up to 20d can you make with these?

What can't you make?

What is the largest amount that you can't make? Explain how you can be sure it is the largest amount that can't be made.

Design your own stamps to investigate (not including a 1c or 1d stamp).

What coins are best?
Teachers' notes

Jane Gabb presents activities on systematic work and combinations in familiar everyday life settings.

1. Best coins

This is an investigation which involves counting different amounts of money. It gives practice in that skill while presenting a problem to be solved.

Some children will need the support of plastic money so that they can physically make and count amounts. This investigation could be differentiated by limiting the number of coins. (Extending it by adding the £2 coin does not add anything to the investigation though, it just becomes a r epetitive exercise once £1 has been reached.)

Coins 1p, 2p, 5p, 10p, 20p, 50p, £1

(Very good pictures of coins can be found by typing 'UK coins' into a search engine.)

Introduction: Explain that they are going to investigate what amounts you can make (and not make) up to £1, using **just one** of each coin. Demonstrate using the 100 square that you can make 1p, 2p and 3p using the 1p and 2p coins. Colour those amounts on the 100 square. Ask: *Can you make 4p? Why not? What else would you need in order to make 4p?* Colour 4p differently on the 100 square (or cross it out). If they need further demonstration continue, other wise they continue.

They use the 100 square to colour in those which they can make.

Plenary questions:

What patterns can you see in the amounts that you can't make? (all have either a 4 or a 9)

What coins would you need to add in or der to make all these amounts? (1p and 10p)

Extension: If you could make up your own coins, what would you choose to make all the amounts up to £1 using the least number of different coins? (1p, 2p, 4p, 8p, 16p, 32p, 64p – this will make all amounts up to £1.27)

2. Giving change

This activity involves being systematic to find all the possibilities before looking to see what patterns there are in the number of possibilities for each amount. (This was originally a GAIM activity.)

Some children will need the support of plastic money so that they can physically make and count amounts. Differentiate by suggesting that some childr en begin with smaller amounts of change, even starting fr om 1p.

Introduction:

Ask: *If you had to give someone 5p change, how many different ways could you do it?*

Write their suggestions down in the order in which they are suggested.

Then ask: *What would be the best order for these so that we could see whether we have found them all?*

Then ask them to investigate: *What about other amounts of change? How can you be sur e you have found all the ways? Y ou can use any coins for this activity.*

Can you find any patterns in the number of ways of giving change?

Plenary questions:

Take in some of their results, checking for systemic orders. List the total number of ways as well as the details of those ways.

What patterns can you see in the number of ways of giving change?

3. Strange stamps

This activity also involves being systematic to find all the possibilities. Pattern spotting also plays a part in understanding what happens with these pairs of numbers.

Introduction:

On a small island the only stamps available are 2c and 5c stamps. Ask what amounts could be made just using 2c stamps. Repeat using just 5c stamps.

Ask: *What if we can use both together? Can you give me some examples of amounts we can make now?*

What amounts of postage up to 20c can you make with these stamps?

What can't you make?

What is the largest amount that you can't make?

A further investigation on the same lines:
On another island the stamps available are 3d and 4d.

What amounts of postage up to 20d can you make with these?

What can't you make?

What is the largest amount that you can't make?

Going on with their own ideas:

Investigate for different pairs of stamps (not including a 1c or 1d stamp).

Plenary questions:
How can you be sur e that your answers (to the largest amount that can't be made) *are corr ect? How do you know there isn't another larger amount further on in the larger numbers?*
Can you pr edict what that lar gest number would be if our stamps were 3p and 5p (for instance)?

Royal Borough of Windsor and Maidenhead.

Munn's the word for maths ace Michael

Under this headline *The Northern Scot* , of Friday December 16, 20, announced to its readers the winner of this year s Harry Hewitt Prize. After missing about 18 months at school where he experienced problems with other pupils and began skipping classes Michael Munns started at Moray Council s Pinefield Project Assessment and Resource Centre in August 2004. Michael s progress was then remarkable journalist Chris Saunderson wrote. Michael is quoted as being surprised at winning the award but said he was really chuffed as well. It just shows you don t have to be good at school to be good at maths, he commented. His teacher, Pat Hall, who nominated Michael for the award said, He absolutely blossomed, which doesn t happen every time but when it does it is absolutely amazing.

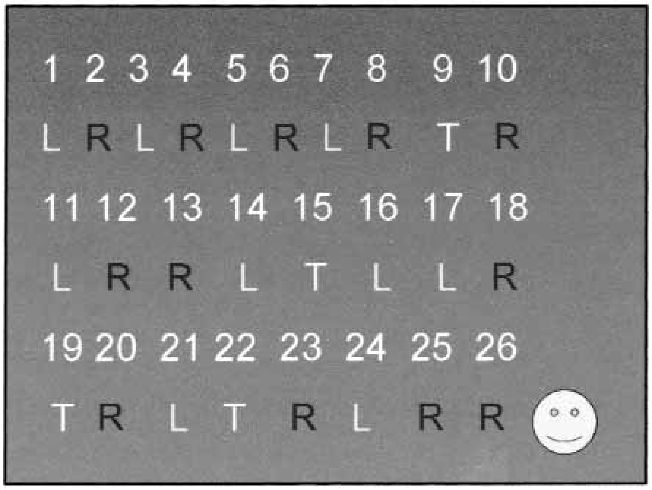
In the Northern Sco t s picture with Michael are from left Bill Richardson of the Mathematical Association, David Cooke the project co-ordinator, Pat Hall, his teacher and his mum Mandy.

If you have a pupil who has suddenly made remarkable progress in mathematics - from however low a level - just write to *Equals* before the end of June describing it and put your pupil in for the 2006 prize.

Wave 3! What's Wave 3?

Patsy Smith (maths coordinator) and **Marjorie Clementson** (SENCO and AST for Inclusion) describe how they put across the ideas behind the Wave 3 pack to their staff, using about 75 minutes in a staff meeting.

We started with a few warm-up and orientation activities:

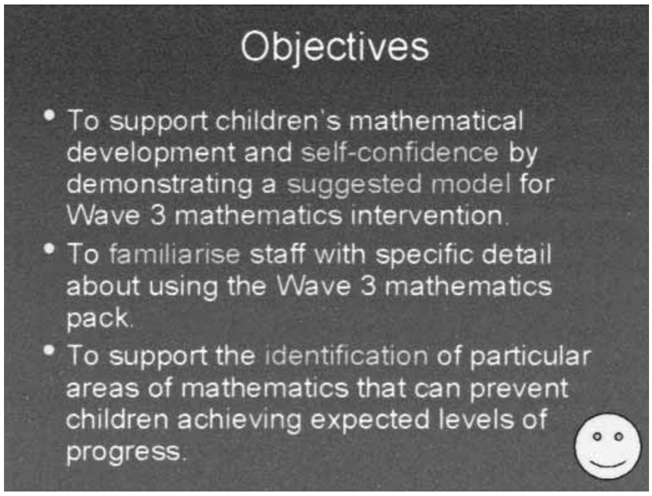


This slide was the first one we showed and we asked staff to participate, berating them (gently) for not remembering what it meant. (Role play as a teacher leading a class of pupils – "Come on", "What's the matter?", "We did this yesterday")
We asked how it felt to be put in the position of children who don't remember and who don't know what's going on.

We then invited them to look at some photographs depicting various emotional states, which we had placed around the room, and asked them to go and stand next to the one which summed up best what they had felt about maths when at school. (The pictures we used were the photocards from the SEAL [Excellence and Enjoyment: Social and Emotional Aspects of Learning] materials, but any photographs showing a range of emotions would be suitable.)

We asked a selection of the staff to explain why they had stood by their pictures, which revealed a wide variety of attitudes towards mathematics. We followed this with a brief discussion on why some children have a negative reaction to maths.

The activity on the slide above was then re-presented and it was explained that L meant 'tap your left shoulder with your left hand', R meant the same with your right hand and T meant both together. The activity was repeated with everyone participating.
This slide was used to explain the objectives of the session.



We moved on to explain that the session would cover:

- What is Wave 3 mathematics?
- The Wave 3 Cycle
- Wave 3 at our school

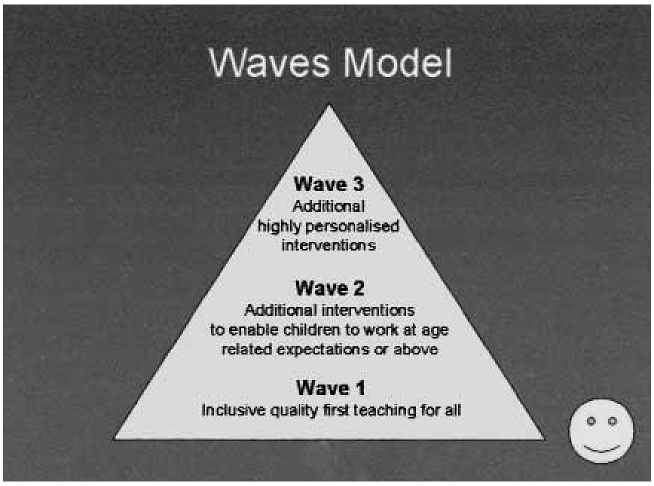
We explained that the Wave 3 mathematics box was entitled: 'Supporting children with gaps in their mathematical understanding' and explained why: that the aim was to address fundamental errors that prevent progress.
The Wave 3 Mathematics materials focus on number and calculation, tackling areas such as understanding the structure of number and operations between numbers. Application and problem solving are also included.

We identified the target population as:

- Pupils below level 2 at end of KS1
- Pupils below level 3 at end of KS2

and explained that the Wave 3 pack could be used to assess, track back and teach using focused practical activities. Gaps are identified and then specific teaching activities are used to bridge these gaps and enable the child to build on their knowledge and understanding.

We showed this diagram to explain where Wave 3 fits into the programme for mathematics:



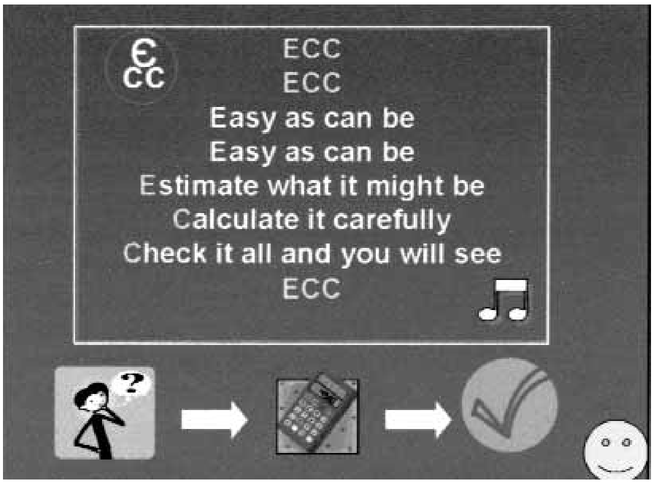
We then asked: 'What do you think are the most common gaps in mathematical understanding?' and invited staff to write on post-it notes which we then collected and put on a flip chart, sorting them into common categories.

Many of the ideas which appeared on our flip chart covered what is in the Wave 3 pack namely:
• Ordering numbers
• Counting on and back
• Partitioning and re-combining
• Addition and subtraction facts with 20
• Understanding the 4 operators
• Providing problem solving activities

After this we went through the cycle of assessment and intervention which is described fully in the booklet 'Using the pack' which comes with the Wave 3 materials. Examples of the tracking charts were on the tables and some time was spent going through these to explain what it all meant.

We identified these as fundamentals:
• Using and applying mathematics are integrated.
• Development is emphasised and key vocabulary listed in each activity.
• Focus on progression in counting.
• Emphasis on the process of estimating first, then calculating and then checking.
• Decimals are addressed within meaningful contexts.
• Structured equipment and everyday materials are used to model mathematical concepts.
• A wide range of resources is used in teaching sessions.

In order to introduce the symbol which appears whenever estimating, calculating and checking are suggested, Patsy invited everyone to join in a song which she had written to help children understand and remember about this. The song is sung to the tune of 'Three blind mice' and we all joined in.



Now we provided an opportunity for everyone to spend some time looking at the materials. (We had prepared a few of the booklets with the resources necessary for the activities and put them on tables labelled R (Reception), year 2, year 4 and year 6. It had been explained that the 'Year' referred not to the child's actual year group, but to the stage of their mathematical understanding.) For many people this was felt to be the most important part of the session and we recommend at least 15-20 minutes is put aside for this. It is also a good time for the presenters to circulate and answer any questions.

When we got back together we said that the key messages were:
• Wave 3 materials are a suggested model which can be adapted.
• They aim to support good practice in working with children.
• Good day-to-day assessment is fundamental in supporting effective use of Wave 3 intervention materials.

After this it was time to discuss how Wave 3 might work in our school. Some of the aspects to discuss were:
• Overall leadership and co-ordination.
• Assessment and recording.
• Who? (teacher & pupil).
• When?
• Where?
• Frequency?
• Storage of and access to resources.
• CPD for everyone.
• Opportunities to involve parents and carers.

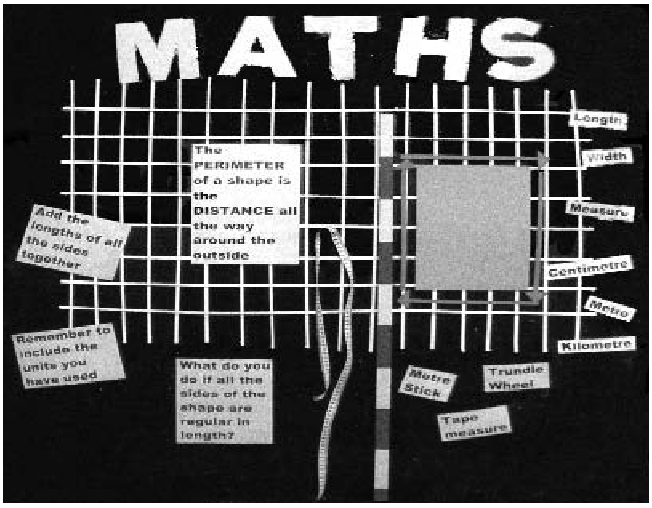
Furze Platt Junior School
Royal Borough of Windsor and Maidenhead

Using the Digital Camera

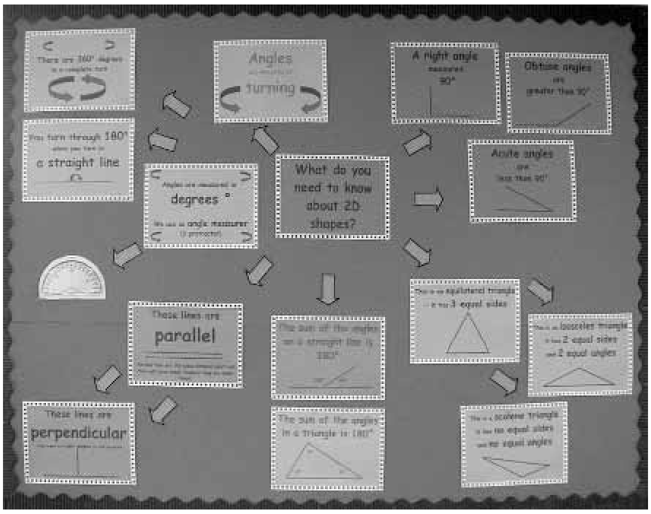
Jennie Pennant starts us off on some mathematical adventures with the digital camera.

Over the past few years we have seen the arrival of the digital camera and, more recently, the digital camera as an integral feature on many mobile phones. These cameras are now both readily available and inexpensive. Thus it is practical to consider their use in classrooms to promote mathematical learning. Outlined below are some ways in which the camera may be useful to encourage children to make links in their learning and to talk about their learning.

Memory prompts.



Area and perimeter



Angles

We all know how useful a relevant wall display can be to the children's learning. We refer to it in our teaching and see the children looking back at it to clarify or recall information. Prior to the advent of the digital camera these wall displays have had to be taken down after a limited period, often around six weeks. Now we find we have the technology to photograph them. This opens new opportunities for their use. We can use them for recall when we next meet the topic or we can use them when we are revising a topic. Either way they support children's mathematical recall and also mathematical talk. In addition we may consider sending them on with the class to their new teacher in September. This offers the next teacher the opportunity to show the children some familiar resources when they first explore that topic together.

Making learning real and relevant to the pupils

Shoes twos

Young children, especially, are very captivated by themselves and their world. With digital photography we can use this interest to promote mathematical learning. For example, children readily understand that shoes come in pairs – one for each foot.

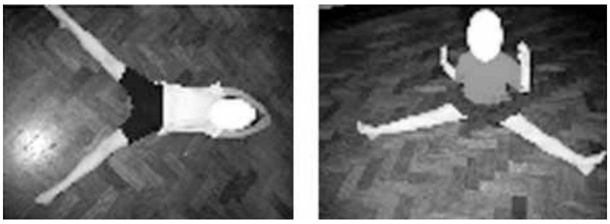
The image of shoes helps to associate the idea of counting in twos with knowledge that they already have and it enables them to build a mathematical model of counting on a real life picture. The advantage of counting in twos is clear as each shoe has its own partner and as we count a pair we need to know that it is two shoes.

Taking this idea further, a photograph of the class with their hands stretched out waving in front of them, at shoulder height, can be a useful visual image on the interactive whiteboard, for counting in fives and for questioning such as,

' How many children will I need to count 30 fingers?'

‘ If 10 children put their hands behind their backs, how many fingers would disappear?’

Bringing the mathematical learning from outside the classroom into the classroom



Symmetry PE

Playground compass

For some children making the links between their mathematical experience outside, or in another area of the school, and what is happening in the mathematics classroom can be a challenge. We all know the value of exploring symmetry in the PE lesson and offering the children the opportunity to feel and see what it is like using their whole body. This could be seen as an extension of learning to count with our fingers - ‘do it, feel it.’ Now, using a digital camera, we have the opportunity to photograph these symmetrical images that the children make and bring them back into the classroom for display on the interactive whiteboard. Thus we can make the links more clearly. The children are no longer left relying on their memory and the teacher’s auditory prompts to remind them of their experience. In reviewing these pictures back in the classroom, we have an opportunity to promote mathematical talk as the children hear and practise the accurate use of key mathematical vocabulary. As the children explore their experience, we can make the links with the more diagrammatical models that we use to show symmetry. Depending on the angle of the picture, it may be possible to use the interactive whiteboard tools to draw the outline over the symmetrical body shape. This can then be pulled off to a plain part of the screen, thus showing the connection between the real life example and the more diagrammatic example of symmetry. It is important that the children see where these diagrammatical models come from.

Playtime and the playground offer a wealth of opportunity for mathematical experiences. Images painted on the playground from hopscotch to large size number squares can provide the basis for mathematical play and exploration. A digital photograph of one of these, such as a large size playground compass, can be displayed on the interactive whiteboard in a lesson on direction The teacher is now able to use this as a basis for stimulating a discussion on direction and encourage the children to share their ideas. It is again strengthening the link to their own experience and helping them to embed their mathematical learning in their whole learning context.

Using pictures of the local environment to promote mathematical learning.

Shop window sales

A real feature of the digital photograph and its display on the interactive whiteboard is the facility to bring images of the outside world into the classroom. The digital pictures give the opportunity to make the connections between their mathematical learning and real life experiences. How often do we refer to the fact that percentages are very useful if you want to calculate the sale price of items? Now we can show examples of that not just as an illustration in a textbook but as a current picture from the children’s own locality – the high street that they are familiar with. This contributes to making the learning relevant to them, thus promoting connections in their minds.

Setting mathematical learning in a real life context

Well, what is the point of it?’

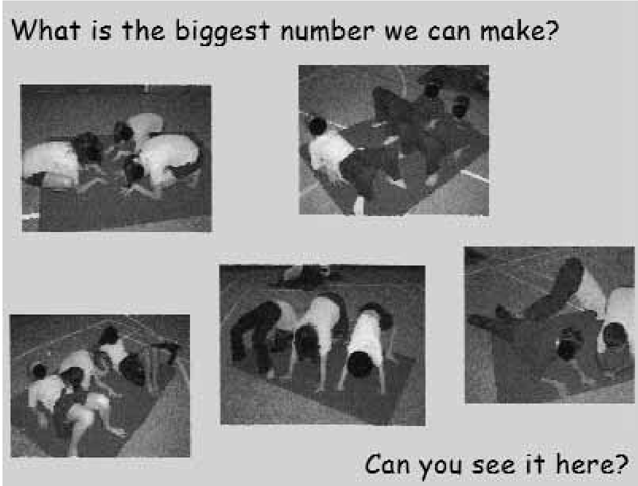
This is a familiar question that children ask. Our ability to answer this question can be assisted by the display of digital photographs that show a real life use of the mathematical learning.

The conversion of imperial to metric units can be just such a challenge, yet the photograph of the bridge shows a practical application of it, again in the children's locality.

Conversion imperial to metric

Recording outcomes of a piece of work in an alternative way.

'Hands are worth 5, feet 10, elbows 2 when they are in contact with the floor. In groups of three, using at least one elbow, foot or hand from each person, can you make 57?'
This is the start of an interesting investigation into which numbers it is possible to make and which, if any, are impossible. One of the challenges of this type of investigation is the style of recording. Digital photography means that the outcomes can be recorded as a snapshot. These can either be used as the end product or as a discussion starter to model alternative ways of recording, such as a table.



Problem solving activity

Conclusion
The advent of digital photography, at a cost that makes it readily accessible, has brought with it the opportunity to foster links to promote children's learning. There is a range of possible links that can be encouraged, as detailed out above. The display of digital photographs also provides a stimulus for mathematical talk. The children can be encouraged to talk about what they see and learn to use mathematical vocabulary accurately as they explain. Both the fostering of links and the promotion of mathematical talk can contribute to the children's ability to grasp and internalise mathematical concepts, helping them to make sense of these in their own world.

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Pages from the Past

An idiot teacher sees no problems with learning

We continue with the thoughts of **E. F. O'Neill** [1] and his beliefs about education. His views on marking children's work chime with recent research and today's understanding about Assessment for Learning.

Teddy's 'Credo'
'I believe that education should deal in realities and not be artificial. It should be concerned with the day's work of the Jack-of-all-trades, children and teachers, the response to actuality, genuine employment called for by the circumstances of their environment, inside school as well as outside.' ...

'I believe that the *ability to find out* and the desire to do so matter rather than any limited load of information a child can carry remember, and repeat.' ...

'I believe that the function of a lesson should be to provide opportunities for the exercise of the life force latent in every child and to facilitate such exercise in every possible way, and never to withhold opportunities,' ...

'I believe that teachers should do things *with* children rather than *for* them.' ...

‘I believe that children should be allowed to work together, to discuss their work one with another, and to learn by helping each other.’ ...
‘I believe that these yards and buildings are educational laboratories.’

‘I do not want you to ask me what they know. Ask me: How are they growing?’
‘What kind of people are they becoming? Are they going to be able to fit in and be effective elsewhere?’ ...

First then, as regards literacy and the ability to calculate. These, the three R’s, ... are classed as primaries. They must be exercised every day before the children undertake any self-chosen activity at all. They follow a definite and well considered programme, but they follow it at their own rate. They have to accomplish a minimum amount of work in these subjects, but the atmosphere of activity in the school secures that this minimum is not regarded as a maximum: they may go ahead, the programme has no end, nor has the time available any fixed limit. ... the aim here is that, by being given abundant facilities to use their inborn powers of discovery and interpretation, these faculties will grow and grow through such healthy exercise. The knowledge will accumulate as a by-product of this activity, but initiative and resourcefulness will have greatly increased. ...

Of all the incentives, that known as ‘marks’ must be the wickedest, stupidest and cheapest. Cheap it is, without question: marks cost no one a penny. Stupid, because it forces false values on the child’s conscience and urges him to do the job, not for the joy of doing it, but for worthless gain. Wicked, because the mark system forbids the exercise of the natural virtue of helpfulness. It is natural and advantageous that children should discuss with each other ways and means of resolving difficulties. They are thinking mathematically and learning from each other and arguing as to whether or not they either or both reached a correct solution. The marks system, forbidding collaboration, which it would class as cheating, opens the way to cribbing. Even more lamentable, the marks system asks the child to compete with his classmates for these prizes and tempts him to regard his friends as rivals and possible enemies; to pride himself on his petty achievements and to look down on those who he may happen to surpass. ... Incentives, motives put into the mind to induce a worker to do some job, if they are extraneous to the work in hand, whether they are prizes or punishments, should find no place in a well-considered scheme of education.

All right: all these children work regularly every day at these primary subjects before they do any self-chosen work. If they have not been contaminated by working under a marks system before they come to O’Neill, and have been under his care from infancy, they will be found to be enjoying their study of primaries for its own sake, performed as it is in an atmosphere of freedom, initiative, and discussion. Moreover, not having been brought up to be dependent on a teacher for guidance and instruction, their faith in themselves will not have become paralysed.

Two of the Proverbs of Teddy O’Neill: More proverbs in later issues

The problem of education is the idiot teacher:
For whom no problem exists
Who expects children to do what he himself can’t – learn
Who can only do what he has done
Who only wants to teach his own subject
Whose qualification is that he has passed his exams
Who is repetitive and uncreative
Who has never really lived
Who has a bus to catch

A school should be:
A place for lectures and teaching
A workshop for young and old – of both sexes
A den of hobbies and indoor games
A studio for drawing painting and plastics
A music studio
A hall for song and dance
An educational shop window
A reference library
A picture gallery
A museum
A reading room
A book-stall for magazines and newspapers
A club
A place for parties
A refreshment bar
An orchard
A zoo
An aquarium
A vivarium
A home for pets
A playing field
A gymnasium
A bathing place
A fair garden
A kitchen
A dining place
A laundry
A first aid post
A cleansing department
Store sheds for raw materials

1. E.F. O’Neill of Prestolee as recorded by Gerald Holmes London: Faber and Faber, 1952

Combined Operations

Stewart Fowlie suggests support activities for pupils who have difficulty in recognising generalisations in simple arithmetic.

As children learn to add, they quickly come to realise that, for example, 2 + 6 is the same as 6 + 2, and if you are counting on your fingers, or with a number line, it's much easier to start at 6 and count on 2 than to start at 2 and count on 6. As they learn to add symbols rather than objects, if they read 2 + 6, they will look for the larger number, 6, and add the 2 to it.

When they move on to subtraction, they may be asked orally to take 2 from 6, or what 6 take away 2 is but in writing they will always be asked 6 - 2. It is not surprising that they think that means take the smaller number from the larger, and 2 - 6 would mean the same thing.

In multiplying two numbers together, it is usual to teach multiplying by the smaller numbers first, and work up. Children may be asked what 2 times 6 is, or what 6 multiplied by 2 is. In writing they may be asked 2 x 6 or 6 x 2. Usually they will focus on the larger number and multiply that by the smaller.

In division as in subtraction, the children generally meet calculations where larger numbers are always acted on by smaller numbers.

In no case has the child any need to consider the order the two numbers appear in: the larger number is always acted on by the smaller.

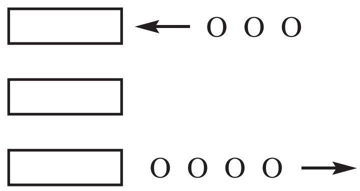
While children early on meet situations involving adding several numbers together, and will understand and be able to work out for example 3 + 7 + 4 + 8, usually questions involve only two quantities, which have to be combined to find a third quantity. More complicated questions are split: for example:

> There are 7 children in a room. 3 more children come in. How many children are in the room now?
> 4 children then leave the room. How many children are in the room now?

Consider this question based on the same situation.

> There are some children in a room. 3 more children come in.
> 4 children then leave the room. How has the number of children in the room changed?

At first sight this might appear to be another typical question involving two quantities. To the child it seems to involve three, one of which is not revealed. In fact two operations have to be combined to produce a third operation. Some children may be able to find the answer, but many will say they are stuck because they haven't been told how many children were in the room to begin with.



The first picture shows the room with the 3 children waiting to go in. The second picture shows everybody in the room. The third picture shows the room and the 4 children who have cone out. There must be one fewer child in the room.

Whenever a new concept is to be introduced it is essential to start from a concrete situation. Before drawing the pictures your class should act out the situation. Send three outside. They then come in, and then they and a fourth go out.

This can be recorded as +3 - 4 = -1, read as "add 3 and subtract 4 gives the same result as subtract 1", or to begin with as "3 going in and 4 coming out gives the same result as 1 coming out."

Questions can be asked in "fill in the blank" format:

+3 - 4 = _ ,
+3 _ _ = -1, and
_ _ - 4 = -1.

It should be pointed out that operations of this kind are commutative. Notice that all this can be done without explicitly introducing the concept of positive and negative numbers.

Another situation which can be treated similarly is climbing upstairs, and going backwards downstairs. Our problem above would become: "if you go up 3 steps and then down 4 steps, how many steps are you from your starting point?"

The pictures would be easier to draw if you were taking steps forward or backward along a line, or going up and down floors in a lift. Further examples might include putting on and losing weight, and getting warmer or colder.

Edinburgh

Correspondence:

Hi Equals,

Dyscalculia and support systems

My name is Kathryn Hopson I am a senior Library Technician working at Bribie Island Library, Australia. I would like information on a subject that I am trying to educate people on and that is a learning disability recognised in up to six per cent of the world's population and I would be interested in seeing what literature has been produced to help teachers and students to identify this area of mathematics learning.

I have found that America and the UK are leading the way on information. I would like to publish my own book on this topic as there does not seem to be any in depth research done on this in Australia as yet. However I would like to collaborate with the Mathematical Association to find out your research first.

Thank you for any assistance you can give me, please feel free to contact me - email: hopsonk@caboolture.qld.gov.au or

Kind regards,

Kathryn Hopson

NB The editors hope that any responses will be copied to Equals so that any further discussion and information can be published for the benefit of all readers

Students with SLD and autism

Please advise the kind of literature that I can access to support my role as maths coordinator at a special school. High proportion of students working at P level 3 - 8. A few at L1? I would like to carry out a study - post Ofsted report on promoting and developing mathematics across year groups.

Regards

Linda Povey

Where have all the sparrows gone?

Male and female sparrows stay together for life – sometimes for six or seven years. They use the same nest site year after year, if it remains suitable.

There were 2,600 house sparrows in Kensington Gardens in 1925. In 1948 there we 855, by 2000 there were just eight.

Over the last 10 years 7 out of 10 sparrows have been lost from Greater London.

Sparrows have lived in the Underground and have even survived for several years trapped 600 metres underground in a coal mine.

Help us to stop sparrows disappearing before it is too late.

RSPB leaflet. See www.rspb.org.uk/london

Uganda:

Every evening 19,000 'night dwellers' (most of them children) travel into town from the rural areas to avoid being abducted.

Oxfam News Spring 2005

Reviews

A Starter for study:
Anne Watson, Jenny Houssart and Car oline Roaf eds, *Supporting Mathematical Thinking* , London: David Fulton in association with NASEN, 2005
introduced by Rachel Gibbons

The book aims to offer guidance to teachers and teaching assistants for supporting pupils with dyslexia or dyscalculia, autistic spectrum disorders, speech and language difficulties, hearing impairments and visual impairments. It has such a wealth of good ideas and collected wisdom that it seemed appropriate, rather than review it briefly, to note some of the ideas and use them as the basis for suggestions of ways to work in your own classroom - whether your pupils have the difficulties listed above or others - and then write about what you have done in future *Equals*. The advice would prove useful to all teachers. Because, as the book (quoting from the Warnock Report) reminds us "the purpose of education for all children is the same; the goals are the same",[1] we believe the analysis the contributors make of children's approaches to mathematical ideas and ways of responding to their different points of is of value to teachers in all institutions working with all varieties of pupils. The contributors describe elements of the best teaching practice while noting that different pupils may well need different approaches because " the help that individual children need in progressing towards [their goals] will be different".[2] First, the need for a long term and continuous process of development for teachers is established, which includes "space for research and reflective practice in collaboration with others",[3] so that both teachers and pupils can

"• become confident in their own mathematical resources and in their power to understand, use and generate mathematical ideas
• extend their own knowledge by developing their own skills of self-teaching and enquiry, and
• learn to value their own and others' expertise."[4]

To engender innovative thinking five moves are suggested:

"• making connections,
• contradicting,
• taking a child's eye view,
• noting the impact of feelings and
• postponing judgement in order to find out more".[5]

This is only the beginning of the collected wisdom presented and it seems to have enough to go on for at least the first year of a co-operative association between a group of teachers from neighbouring schools who are keen on improving their own practice and extending the opportunities of innovative mathematical thinking for their pupils.

The authors put forward their belief, which is indeed the very reason for the existence of *Equals*, that "the craft of teaching cannot be developed in isolation". Therefore, before making any further comments on this influential volume we challenge you to give yourself some space by getting a group together for research and reflection on your own practice. Looking at the five suggestions above in a little more detail will enable you to try out ways of putting into play in your own classroom some of the "thinking moves" listed above, and to discuss them with others:

What happened when you (or a colleague) did:

make a connection for a child
One suggestion given in the book is : "take a blank page and write any number in the middle of the page. Now fill the page with different ways of thinking of the number – use addition, subtraction … Be imaginative!"

contradict a child
How did you do it?
did you say, for example, "No that is not a rectangle, do you know why it isn't?"

take a child's point of view
To see the way pupils approached some simple calculations they were asked to explain their strategy, e.g. 22 – 17
"I did it with bricks. I take 22 and I take 17 from the 22."[6]

note the impact of feelings
The 'one right answer' characteristic of some mathematics can engender anxiety.
Rosalyn Hyde suggests that the use of graphic calculators (and this is true of all electronic devices used individually) "is non-judgemental – you can get it wrong and no one knows but you." [7]

suspend judgement while listening to a child's ideas
In describing instances of responsive questioning Mundher Adhami writes, "All answers are acknowledged as valid contributions." [8]

Now that PPA time is being built into everyone's timetable it should be more possible to plan, prepare and assess at a deeper level. What better way of analysing our students' progress, planning exciting mathematical pathways for them and preparing materials for their guidance could there be than sharing your insights and hammering out mathematical and pedagogical ideas with others from a variety of educational establishments and stages of teaching? Even only two teachers in one school working on these matters together can have an amazingly beneficial effect on their own and their pupils' progress.

Finally, to consolidate learning it is always useful to write reports of what you have learned. We therefore challenge you to write up some of your joint findings and send them to Equals for wider circulation.

Fulham, London

1.page 3
2.ibid
3.page 5
4.page 9 – quoting from Ahmed and Williams, *Raising Achievement on Mathematics Pr oject. RAMP Report*, West Sussex: ChIHE/HMSO, 1992
5.page 5 - quoting from Hart, *Thinking Through Teaching*, London: David Fulton, 2000
6.page 25: Harries, Barrington & Hamilton What do children see?
7.page 99
8.page 138

Review by Jane Gabb
Getting the Buggers to add up
Mike Ollerton
Continuum
ISBN 0-8264-6879-9

Firstly, don't let the title put you off this book! It is part of a series with similar titles:
Getting the buggers… to behave, to write, to draw and into languages.

Mike Ollerton's greatest strengths are that he has a real love of mathematics and a passion for teaching mathematics to adolescents. When he was a head of department mathematics was taught to mixed ability groups in ways that did not involve using textbooks or any published scheme. His approach was through equipment-based problem-solving. He describes the book as 'an attempt to offer alternative visions and different ways of teaching mathematics.'

The criteria he uses when planning mathematics through problems are to:

1. Provide access to students in terms of everyone being able to offer some kind of answer.
2. Gain a variety of answers or encourage different ways of working on an idea.
3. Help students move towards an understanding of a specific concept.
4. Cause students to work on ever-more complex areas of mathematics.

His chapters on mathematical surprises, people-math and mental maths provide some very rich ideas for the classroom, including work on shape, fractions, properties of numbers, place value and loci.

He devotes a chapter to the use of practical equipment, so important for engagement and helping understanding, particularly, but not exclusively, in those who struggle with mathematics. Other chapters concern 'real-life' contexts, ICT and thinking skills. He also addresses how to plan a scheme of work, teaching without a textbook and managing the mathematics classroom.

With such a wealth of material it is difficult to choose an example to illustrate what this book has to offer. However I found his exploration of the mathematics to be found by using a 9-dot geoboard fascinating. Pupils could begin in a very practical way by finding all the possible shapes that can be made with an elastic band on the 9-pin board. These can be named and classified and their properties explored. Congruent shapes can be identified while ideas of reflection and rotation appear naturally in the course of the work. The shapes can be drawn on large grid paper, so that their angles can be explored. Areas can be calculated and perimeters measured. (There are 16 possible quadrilaterals and 8 triangles, not counting reflections and rotations). Pythagoras' theorem can be used to calculate the diagonal lengths and students can practice trigonometry by working out angles. This illustrates very well the progression and differentiation possible when using practical equipment and a simple starting point which provides access for everyone.

In short, this book should be essential reading for all mathematics teachers, particularly those who wish to free themselves from the constraints of the textbook, setting and the 'teaching to the test' culture. It is reassuring to note that when Mike Ollerton took this approach with his department examination results went up and continued to rise.

Royal Borough of Windsor and Maidenhead.