Equals

mathematics
and
special educational needs

for ages 3 to 18+

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MATHEMATICAL ASSOCIATION

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Equals

mathematics
and
special educational needs

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Letters and other material for the attention of the Editorial Team to be sent to:
Rachel Gibbons, 3 Britannia Road, London SW6 2HJ

Equals e-mail address: equals@chromesw6.co.uk

Published by the Mathematical Association, 259 London Road, Leicester LE2 3BE
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Editorial

If you are old enough you will undoubtedly remember where you were when President Kennedy was assassinated or when England won the World Cup or Neil Armstrong set foot on the Moon. These momentous events seem to transcend everyday events to such an extent that the image of the moment always stays with you. In recent times, the planes hitting the Twin Towers in New York was undoubtedly one of those moments. I can recall where I was vividly because I was on my way to a meeting of the *Equals* editorial team and was listening along with several other people to the events on a radio on a newsstand outside Russell Square Underground Station. The memories of those events were brought back just recently when Russell Square was one of the tube stations attacked on 7th July in the first of the suicide attacks on London's transport system. Many words have been written since about the mentality of those who perpetrated this atrocity but it is still no easier to understand or condone in any way. But maybe it is relevant to question whether these young men have felt 'included' in the education system of this country.

One of the debates following the London Bombings has been about how individuals actions are influenced by the society in which they live. Although on a different scale the problem of truanting is one where the individual, or the individual's parents, are the ones seen to be in the wrong. However, Rachel Gibbons takes a different stance blaming the 'society' of the classroom and the curriculum for pupils wanting to truant. When you think about it, we, as mature adults, would find life very difficult if every 45 – 60 minutes we had to change what we were doing, even if we hadn't finished, to go to a different environment, not of our own creation, to be taught an entirely different discipline by an entirely different individual and to be expected to behave all the time! There have been many people brave enough to stick to their convictions to attempt something different. Pages from the Past recounts events from the early part of the 20th Century and Corinne Ainger and Hilary Povey relate what can be done even within the confines of the school timetable if you are prepared to let pupils have a voice and listen to them!

We have three articles on the teaching of arithmetic in this edition of *Equals*. Ian Thompson challenges the link between approaches to adding and subtracting numbers ending in 9 and 1 as suggested in the *Framework for Teaching Mathematics: Reception to Year 6*. Martin Marsh and Tandi Clausen-May both consider the teaching of division an area that causes so many problems for children because division 'sums' are usually done without helping pupils to understand what is really happening. Martin discusses the importance of teaching both sharing and grouping and Tandi looks at the importance of developing a mental image of division.

We are delighted to publish a piece of work submitted for the Harry Hewitt Prize. You may recall that Harry was an editor of *Equals* and its predecessor *Struggle* for many years and was dedicated throughout his professional life in helping pupils overcome difficulties in mathematics. We are always keen to receive material that demonstrates a pupil working with their teacher to overcome a difficulty and will give prizes to the pupils whose contributions we publish.

Our Centre Spread is about big numbers. At the time of writing the football season has just started (did it ever finish!) and millions of pounds for football players are being tossed around like confetti. Thank goodness, at least at the time of writing, our cricketers are pushing football off the back and front pages of our newspapers. We also share with you further ideas and activities from the *Equals* session at this year's BCME.

And so we come to the end of 11 years of publishing *Equals*. By the time this is read the new school year will be well underway and the *Equals* editorial team will be travelling via Russell Square station to meet to put together the first part of Volume 12 for publication after Christmas. We are always looking for interesting articles about anything to do with mathematics education. Don't forget, if you try something interesting in the classroom there are bound to be hundreds of others who would like to try it too. Write it up and let us have it!

Happy New Academic Year to all our readers

In the last edition of Equals we forgot to mention Mark Else's school at the end of his article. It is Boroughbridge High School, North Yorkshire. The editorial team would like to apologise for this omission.

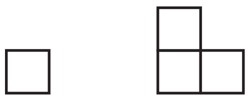
Equals at BCME (continued)

In the last issue we showed you a couple of the activities produced at our session at BCME. Here, some more of the ideas are presented.

1. Growing patterns

Objectives
Primary Framework: 'Solve mathematical problems or puzzles, recognise patterns and relationships, generalise and predict'.
KS3 Framework: 'Generate sequences from practical contexts, finding the nth term.'

Take 2 shapes to start e.g.



Ask pupils to continue the sequence according to a rule…… pupils choose their own way of continuing the sequence and then have to explain what they have done as they are asked:
'What's your rule?'

Observe:
- Children's use of equipment to support thinking
- Choice of recording (what range do you encourage?)
- Children's predictions
- Children's ability to generalise

2. 2D or not 2D?

Objectives
Primary Framework: 'Visualise 3-D shapes from 2-D drawings.'
KS3 Framework: 'Use 2-D representations to visualise 3-D shapes'.

Model the activity and show how to use isometric paper to draw 3-D shapes – make sure the paper is orientated correctly.

Join 3 (multilink or similar) cubes together in as many different ways as possible. All cubes must stay on the table.

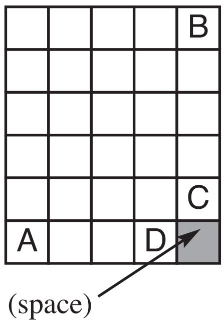
- Draw your shape on isometric paper
- Now try with 4 cubes, draw shapes and show results to your teacher

Possible next steps:
- move on to 5 cubes, all touching table
- suggest that some cubes now no longer need to touch the table. Record.

3. Cube Moving (like missing tile plastic puzzles)

Objectives
Primary Framework: 'Solve mathematical problems or puzzles, recognise patterns and relationships, generalise and predict'.
KS3 Framework: 'Generate sequences from practical contexts, and describe the general term'

Demonstrate the game by using a grid of chairs (spaced) and appointing a director (to organise who moves) and a counter (to count the moves made).



How many moves will it take to get the person at A to move to B? At the start the only people who can move are those at C and D.

Agree that it is difficult to work out.

In small groups find minimum moves in a 3 x 3 or 2 x 3 grid.

What about 3 x 4?
3 x 5?

Move to alternative representation on white board or using Velcro or pencil and paper.

4. Prime factors

Objectives:
Primary Framework: 'Find pairs of factors, recognise prime numbers'.
KS3 Framework: 'Recognise and use factors and primes.'

Equipment: Large 1-30 number grid (each cell to be about A5 size)
Lots of multilink

Activity:
1. Each child takes a handful of multilink. Without counting, ask them to make a rectangle (all cubes on the table)
2. Now count the number of cubes
3. Place completed rectangle in the appropriate number of the number square
4. When multilink runs out discuss with the class which rectangles can be discarded (duplicates)

Questions for discussion:
- Which cells have more than one rectangle?
- Can we find rectangles for the empty cells?

This activity provides a practical lead into Erastosthenes' sieve!
For an explanation of this activity go to:

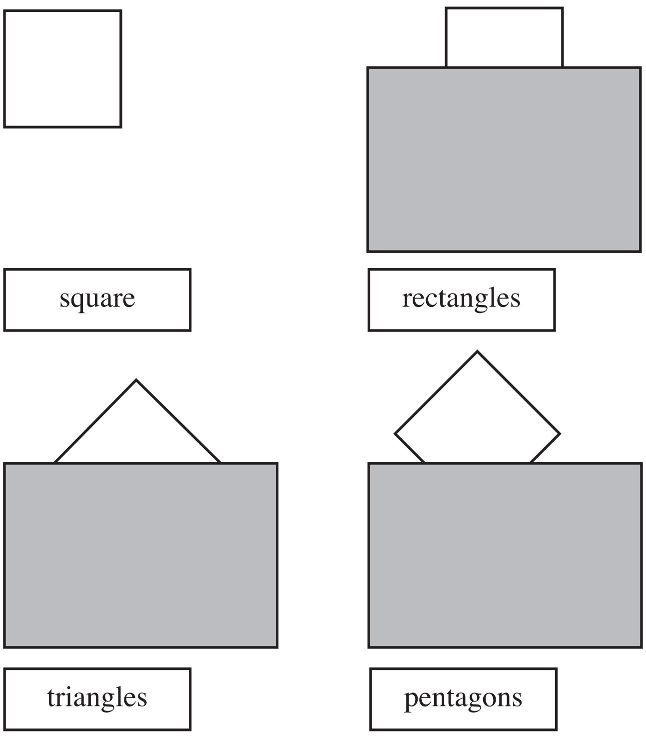
http://www.math.utah.edu/history/eratosthenes.html

5. Hiding shapes

Objectives:
Primary Framework: 'Classify 2-D shapes', 'Solve mathematical problems'.
KS3 Framework: 'Use correctly the language of lines, angles and shapes', 'Solve geometrical problems using properties of shapes'.

Start with a square of card or stiff paper and hide it behind a sheet of paper (laminated paper provides a good slippery surface).

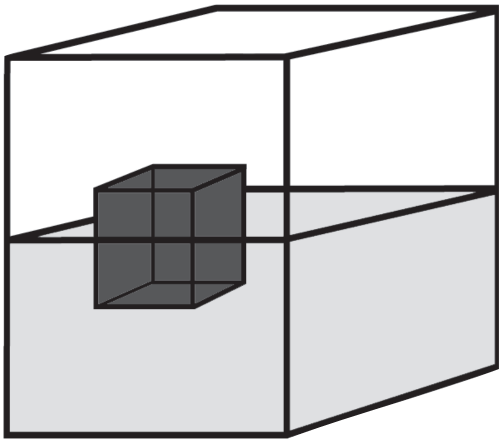
- What new shape do you see?



- What shape is hidden behind the paper?

Questions at different levels:
- Can you sketch the shape?
- What do we call this shape? (Easier – square, rectangle, triangle; harder - pentagon, trapezium.)
- What can you tell me about this shape? (Number of sides and angles, equality of length of sides, nature of angles – right, acute, obtuse, estimate of size of angle)

Extension
More able can move from using the screen and start from a drawn square can draw lines to partition it into different shapes. How many different shapes can you find if you just draw one straight line to cut the square into 2 pieces?



Next lesson introduced by:
A cube in coloured water.
What shape is cut by the water?
Use multilink made into a cube (say 4 x 4 x 4) and cut it to show that the shape cut by the water is a square.

How many other ways can you get that surface? (Rotate by 90°)

Extension:
What other shapes can you get?

We would be interested to hear from any readers who try these activities out; this would also provide useful feedback for those who participated in the workshop. Let us know of any adaptations you made and tell us how your children got on with the activities.

6. Finally…. Music and mathematics.

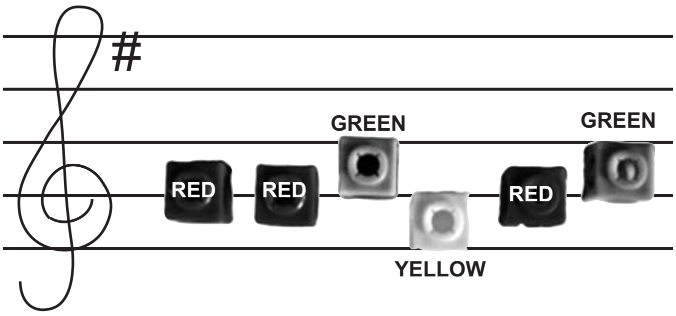
One group at our session at BCME was inspired by the idea of combining mathematics and music from this initial stimulus:

Cross Curricular Links ?

Music: Using different coloured cubes to represent Pitch or Rhythm.

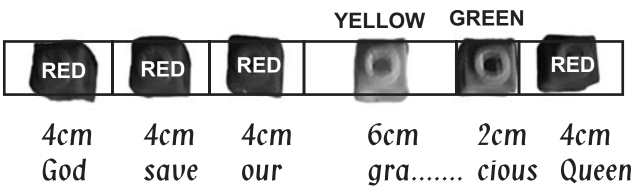
PITCH:-
5 colours for the Pentatonic scale,
8 colours for the normal octave,
12 colours for the chromatic scale.

If red was a 'G', green was an 'A' and yellow was an 'F sharp'; 3 reds, 2 greens and a yellow could be the first 6 notes of the British National Anthem.



Pitch :- Red = G
Green = A
Yellow = F $^{\#}$
God save our gra..........cious Queen

RHYTHM:-
Let colours represent different lengths of note times. If red was a 'crochet', green a 'quaver', yellow a 'dotted crochet' (one and a half crochets), then 4 reds, a green and a yellow would represent the timing lengths of those first 6 notes of the British National Anthem.



Rhythm :- Red = Crochet (1 beat)
Green = Quaver ($\frac{1}{2}$ a beat)
Yellow = Dotted crchet (1 $\frac{1}{2}$ beats)

Pupils can be given known music to 'translate' into coloured cubes, or given assorted sets of cubes and asked to write their own music.
(If teachers do not like the idea of noisy 'rehearsals' as pupils try out their ideas, then computer music programmes and headphone sets could be utilised.)

Eastenders

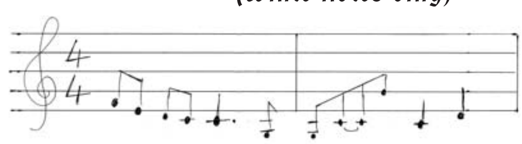
1.Put colour coded stickers on keyboards (Just 1 octave) white notes only.
2.Give students sheet with individual notes to play from the beginning of Eastenders theme tune (no length indicated)

End of lesson 1

Lesson 2

- listen to Eastenders' theme tune
- play tune from previous lesson as written
- discuss difference
- introduce new sheet with length of notes indicated by multiples of same colour.

EASTENDERS :- first 4 bars in C Major.
(white notes only)



Over to you:

We would be interested to hear from any readers who try these activities out; this would also provide useful feedback for those who participated in the workshop. Let us know of any adaptations you made and tell us how your children got on with the activities.

The Idiot Teacher: a Book about Prestolee School and its Headmaster E. F. O'Neill [1]

E. F. O'Neill of Prestolee was an inspired and inspiring educationalist. His wisdom was recognised by the inspector of elementary schools, Edmond Holmes. He still has much wisdom to impart to us today.

Before going to Prestolee. Teddy O'Neill applied to, and was accepted by, a school at Knuzden. One day a strange man came into the school. He stumbled over a boy's legs in coming but the boy did not notice: …he was reading and he kept on reading. The school was alive with activities and the man was bewildered. He was one of His Majesty's School Inspectors. He was used to being shown time-tables and schemes of work and record books and to find himself surrounded by pitch-pine furniture and an atmosphere of silence, perforated by staccato sounds and, presently, to be shown rows of silent up-turned faces. He must indeed have been utterly bewildered. It would not have been surprising if he had sent for a policeman.

As a result of this visit Mr Edward F. O'Neill was invited to Oxford to the Conference of New Ideals in Education to read, on 16 August 1918, an account of Development in Self-activity in an Elementary School. The inspector, Mr. Bloom, then advised Teddy to apply for the vacant headmastership at Prestolee near Farnworth, between Manchester and Bolton.

In undertaking to speak at Oxford he had been forced to attempt some statement of the principles which guided him:

He believed in SELF-ACTIVITY, which he saw as the instinctive doing of things without being told to. Self-active people see what is to be done and do it.

He believed in ORIGINALITY, which he saw as the ability to bring things into being without being a copyist.

Believing, too, in INITIATIVE, he was convinced that the School time-table killed it.
Aware that most people can start a thing but fail to see it through owing to a lack of PERSISTENCE, he believed that it was the school system of set periods of forty or so minutes of lessons, at the end of which children were forced to stop doing whatever they were engaged in doing, which virtually made it impossible for them to develop this characteristic in a healthy manner.

…

He believed that the ABILITY TO FIND OUT and the desire to do so matter, rather than any limited load of information a child can carry, remember and repeat.

…

He believed that every teacher should be a research worker, with his fingers in every pie: never stuck for ideas: never lacking resource, and that in the schools they should find the rich environment which would enable them to live so fully.

He believed that teachers should do things with children rather than for them and children should be allowed to work together, to discuss their work one with another, and to learn by helping each other.

The school was visited by Edmond Holmes, ex chief inspector which led to Teddy being invited a second time to address the Conference of New Ideals in Education…meeting at Cambridge in 1919… Edmond Holmes introducing Teddy used these words:
Mr O'Neill has had effective charge of his new school for only eight months. What he has achieved in those few months borders on the miraculous. If the school, before he took charge of it, was, as no doubt it was, of the orthodox, conventional type, then I can say, without exaggeration, that an entirely new school has come into being. And if I characterise in a few words the change that has been effected I would say that learning by doing has taken the place of learning by swallowing.
No one in England knows better than I do what learning by swallowing means. I inspected elementary schools for nearly six years and during the whole of that time learning by doing was the very rare exception and learning by swallowing was the almost universal rule. …

The children sat in blocks called classes, and opened their mouths like so many fledglings at the word of command, and the teacher then dropped into their mouths pellets of information – rules, definitions, names, dates, tables, formulae, and the like. These pellets of information were as a rule either semi-digested or indigestible, the result being that the young fledglings who had swallowed them made poor growth and seldom found their wings. …
But the child who is learning by doing is learning many things besides the one thing he is supposed to be learning.

The performance of the three R's was still a little akin to the purchase of a railway ticket. It entitled one to proceed.
But, of course, there were genuine enthusiasts even for arithmetic: children who loved doing sums. This, in turn, was useful. If they could be skilfully trailed across an inspector's nose quite a lot might be 'proved'. But this enthusiasm was not altogether satisfactory, nor is an enthusiasm for drinking gin. It would seem when a child sprang eagerly to the problem of simplifying $(1\frac{1}{2} + 3\frac{3}{4})(3\frac{2}{3} - 2\frac{1}{5}) \div (2\frac{1}{4} - \frac{7}{8})$ immediately after the conclusion of Prayers, that his mind must be in an undernourished state bordering on semi-starvation, eager to catch at any straw of interest. …What ? Oh! the multiplication sign. Well, that is a matter of fashion: when you wish to divide one bracket by another you indicate the fact by putting the sign ÷ between them; if you wished to add one bracket to another you put the sign + between them; if you want to take one from the other you put the sign – in front of the one you wish to take from the other; but if you wish to multiply one bracket by another, for goodness sake lie doggo, put no indication of your wishes. What? Never mind 'why': do as you are told, it's quite OK.
Now we come to dividing. Don't do it. It is fatal to try and divide one of these vulgar fractions by another. Turn it upside down and multiply by it. It is all right: do it, and don't waste time arguing about 'why?'. It works, so do it, or we shall get nothing done.
…
Remember all this.
Yes, this is an exercise in obedience, memorising, and faith. … it will result in an increase of that most dangerous idiosyncrasy – blind obedience. And, of course, it tends towards the hypertrophy of memory and the growth of superstition.

'let us use the process of getting information as a tool for developing Character and Intelligence, and let us avoid stuffing children with information for the sake of stuffing him with information or just for the sake of enabling him to pass an examination in that stuffing!' … 'Children must move', he explained 'having to exercise them as a relief from sitting upright for two or three stretches of forty five minutes is as unnatural as forcing them to sit in that way. They should not be immobilised to the extent that they require to burst into violent activity at "play times" and "breaks". Let them move about during their work and fix or change their position as Nature demands.'

Children may work together, for example, and help each other; this is usually regarded as 'cheating' but why? They may even come to quarrelling as to how a sum is to be done, and bring their differences to a teacher for arbitration. What could be better? They are thinking and they are in earnest. … It is this informality of discussion and execution of work which is of educational value, and which formal class-teaching misses.

to be continued….

1. Gerard Holmes, London: Faber and Faber, 1952

Global Warming and Population Growth

Bangladesh, one of the poorest countries in the world, stands to see its population grow by 50 per cent and to lose one third of its territory over the next thirty years. World population will continue to grow until it stabilises in 2030-50 at 8-9 billion people. This is a potential 50 per cent increase on the current world population on a planet already under strain. Ninety per cent of the new people will live in developing countries. The people of the 49 least developed countries, who are the very poorest in the world, currently number 668 million but will reach 1.7 billion by 2050.
Clare Short, An Honourable Deception? New Labour, Iraq and the Misuse of Power, Lonodn: The Free Press, 2004

By 2080, 94 million people around the world will be at risk from flooding every year as a result of global warming; 290 million more will be at risk from malaria. By 2025 two out of three people could lack sufficient water, and drought will cause widespread famine.
And for all this we can blame our runaway use of fossil fuels such as oil. … Esso produces four million barrels of oil a day. Last year their profits were $17.7 billion (£12 billion) more than any other company has ever made. … Esso is currently spending £5.5 billion on oil and gas exploration and production and not one penny on clean renewable energy sources like wind, wave or solar power.
Anita Roddick, Take it Personally: How globalization affects you and powerful ways to challenge it, London Thorsons, 2001

Teaching mental calculation

In introducing some suggestions for improving our teaching strategies, Ian Thompson reminds us that one of the four key principles underpinning the National Numeracy Strategy (NNS) is "an emphasis on mental calculation".

Most mathematics educators would, I think, agree that young children are now generally much more confident and competent at mental calculation than they were before the introduction of the NNS. For example, it is quite common nowadays to observe situations like the following:

- a Reception child finding 'how many beads there are altogether when three more beads are added to the five on the table' by counting on, saying "5… 6, 7, 8";
- a Year 1 child finding 7 + 6 by stating that, "Six and six makes twelve, so it's one more… thirteen";
- a Y2 child calculating 23 - 18 by saying "It's two to 20 and three to 23… so, it's five".

My own research, and that of others, suggests that only a few children made regular use of such strategies before the advent of the National Numeracy Strategy.

In this article I want to take a close look at a calculation strategy that is different from those mentioned above, with the aim of suggesting improvements in the way that this strategy is currently presented in many teaching schemes, and consequently, in the way that it is taught in the vast majority of schools. The NNS *Framework* introduces the strategy in Year 1 and then develops it throughout Key Stage 1 via the objective 'Add or subtract 9, 19, 29… or 11, 21, 31…by adding or subtracting 10, 20, 30… and adjusting by 1'. In Key Stage 2 the strategy becomes 'Add or subtract the nearest multiple of 10 and adjust', only to reappear, renamed, in the context of written calculations as *compensation*.

In connection with the first part of the KS1 objective – the addition of 9, 19, 29… - I would like to mention some research I carried out that involved interviewing 144 seven-year-olds about their mental calculation methods. Interestingly, only one child actually added nine by first adding ten and then taking away one. This research, carried out before the NNS was developed, led me to conclude that the strategy involved was not a 'natural' strategy, by which I meant that it was 'unlikely to be invented by children' (though it was not as 'non-natural' a strategy as decomposition, which, to my knowledge, no child has ever invented!). My findings seemed to suggest that teaching this strategy with understanding might prove more difficult than one would think.

What I particularly want to question about the teaching of this strategy is the wisdom of working on 'adding 9 by adding 10 and *subtracting* one' at the same time as 'adding 11 by adding 10 and *adding* one', as is suggested in all the published materials that I have seen. At first glance it might seem a sensible way of proceeding given the ease with which both procedures can be demonstrated or modelled on the 100-square (see Fig. 1).

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

24+19 =
24+20-1 = 43

56+21=
56+20+1 = 77

Figure 1

However, what usually happens when these two strategies are taught together is that many children (and, in my experience, some trainees) have difficulty deciding which way to move in the second part of the calculation, and this often leads to an incorrect answer. This situation is exacerbated when the *subtraction* of 9 and 11 is introduced later via subtracting 10 and making the relevant adjustment. (Answer quickly: After moving vertically on the 100-square, do you then move to the left or the right when subtracting 9?)

So, when children have been taught the addition and subtraction of numbers ending in 9 and 1 they will have met the following four situations, illustrated here using the empty number line:

Adding 29

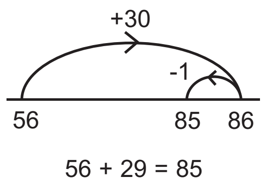


Figure 2

Adding 31

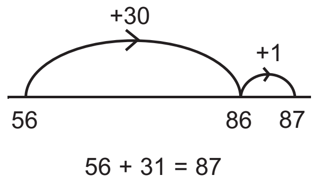


Figure 3

Subtracting 29

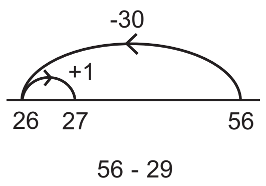


Figure 4

Subtracting 31

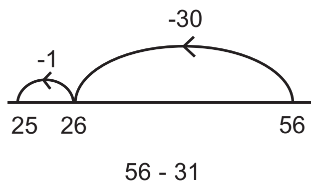


Figure 5

Is it any wonder that children have difficulty remembering which way to jump?

To my mind the main source of the problem is the *Framework*, with its insistence on linking the strategy involved in adding numbers ending in 9 with the strategy for adding numbers ending in 1, whereas in actual fact they are fundamentally different strategies. Adding 21, for example, should be linked with adding 22, 23, 24 or any number in the twenties. It is part of the basic sequential strategy for adding two-digit numbers: when calculating 27 + 24, you partition 24 into 20 and 4; add the 20 to the 27 - giving 47 - and then add the 4 to get 51. On the other hand, adding numbers ending in 9 (or 8) involves a more complex strategy: rather than adding separate parts of the number, as you would when you add, say, 31, you are actually adding a different number from the one you have been asked to add. When adding 29, you do ***not*** partition the number, you add 30 instead, ensuring that you remember to compensate by taking away one at the end of the calculation. This is conceptually a very different procedure.

I believe that this idea of adding a different number demands a great deal of confidence on the part of the children, and that this is probably the reason why only one child used this strategy in my research. My solution to the 'confusion problem' would be to teach the strategies involved in Figures 2 and 4 first of all, as part of the overall strategy for the addition and subtraction of two-digit numbers. This means that there should be no worry about which way to jump when operating on numbers ending in 1 – it's just the same as adding or subtracting any other two-digit numbers.

At a later stage I would teach the addition of numbers ending in 9 utilising the empty number line, and would follow this up almost immediately by teaching the subtraction of similar numbers. Working in this way enables teacher and pupils to focus on the *compensation* aspect of the strategy: after the initial tens jump the child has to compensate for 'over-jumping' by moving one unit in the opposite direction. This compensatory jumping process is the same whether the operation in question is addition or subtraction.

Also, treating adding and subtracting 9, 19, 29… (and even 8, 18, 28…) as a different strategy from working with numbers ending in the digits from 1 to 7 also means that children can be introduced to the correct terminology for the procedure, thus providing continuity with work in Key Stage 2 where a written version of compensation is introduced.

To summarise:

- teach the addition of 2-digit numbers ending in 1, 2, 3, 4, 5… (preferably on the empty number line) by first partitioning the number; adding the multiple of ten; and then adding the units;
- teach subtraction in the same way, subtracting the multiple of ten and then the units;
- when children are familiar with the direction of the jumps involved in addition and subtraction, introduce the addition and subtraction of numbers ending in 9 or 8 by adding the next multiple of ten, and then emphasising the compensation aspect which necessitates a jump in the opposite direction

If the *Framework* ever has to be revised or updated, I hope that the link between adding numbers ending in nine and numbers ending in one is removed, and that the differences between the two calculation strategies involved are clarified.

An earlier draft version of this article appeared in the *Times Educational Supplement*

Northumbria University.

The challenge of poverty

Over a billion with insufficient food, no access to health care, education, clean water or sanitation. Nearly half of the world are very poor, living on less than the equivalent of $2 dollars per day for all their needs.

Clare Short, *An Honourable Deception? New Labour, Iraq and the Misuse of Power*, London: The Free Press, 2004

Grouping and Sharing

Martin Marsh **speculates on the reasons for pupils' difficulties with division. He suggests that an understanding of division solely as sharing has a negative effect, and that pupils can more readily grasp the concept through developing mental images and understanding grouping.**

I was in a Year 9 class recently and a pupil was having difficulty solving the equation

$$3x = 87.$$

Kerry had had no problems up to this point with equations which simplified to equations such as

$$2x = 20, \quad 4x = 32, \quad 5x = 45 \text{ etc.}$$

but the final part of this question was causing her problems.

I went back to the earlier questions and tried to find out

how she had done them so easily.

'If $4x = 32$, I know that $4 \times 8 = 32$ so $x = 8$' was her explanation of this equation.

'Is there another way I could have solved this using division?', I asked

'I know it's 32 divided by 4 but I can't do division' was Kerry's almost instantaneous reply.

Here was the problem with $3x = 87$; she could not find a number which when multiplied by 3 was 87 and as she couldn't do 87 divided by 3 she was stuck.

So here was a girl who was successfully able to manipulate algebraic expressions (the unknown was on both sides before she arranged it) using a balance method (Level 6) but could not do a relatively simple division (level 3).

This got me thinking about the problems that pupils have with division. Why could this girl not 'do' 32 divided by 4? She knew that division was the inverse of multiplication and she knew her multiplication tables. She was clearly quite able in that she was expected to get Level 5 on Key Stage 3 SATs. But she had a real barrier about division.

We can only speculate about Kerry's previous experiences of division, but could they have been something like this?

In year 2 she was first introduced to division as sharing equally although the idea of equal sharing was probably not emphasised. This was probably done quite well using counters or other objects to enable practical approaches to, for instance, calculating $15 \div 3$. Her early experiences were probably very positive but then came a question she couldn't do such as $36 \div 4$ because she didn't have 36 counters to share and she didn't know that $9 \times 4 = 36$.

However, at some stage she learnt that division was the inverse of multiplication and also learnt her tables so was able to get by quite well on simple division questions. 36 divided by 4 was no longer a problem because she knew $9 \times 4 = 36$. She might have been able to do some more complicated division question using a division algorithm such as:

```
      4 3
6 / 2 5 8
    2 4
    ----
      1 8
      1 8
      ----
      0 0
```

However, the algorithm, although taught possibly 4 or 5 times in her time in school never really stuck because it wasn't ever really understood. When she first met long division in secondary school that was far too difficult and she eventually arrived at her current belief that she couldn't 'do' division. Most of the time this isn't a problem but when trying to solve $3x = 87$ it is.

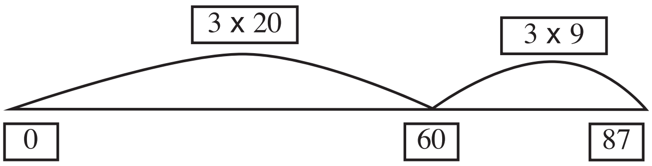
What had gone wrong? Continuing to speculate, I suspect that Kerry's understanding of division was not really much beyond where it was in Year 2 when she was sharing objects. What Kerry did not understand about division was the idea of 'grouping' and how this relates to repeated subtraction.

Let's consider $87 \div 3$.

We could work this out by getting 87 counters and subtracting groups of 3 until there were no counters left and then counting the number of groups of three. This would give us the answer. This is obviously not an efficient method, but when the numbers are smaller e.g 15 ÷ 3, it is as easy as sharing. *The Framework for Teaching Mathematics from Reception to Year 6* emphasises this method when first meeting division as equally important as sharing. (Year 123 supplement of examples p 49)

What grouping does is to lay the foundation of approaches to working out division questions which will be needed later.

Let us return to 87 ÷ 3. Kerry was very good with multiplication. She knew facts like 3 x 10 and 3 x 20. An understanding of grouping and repeated subtraction together with a visual image might have helped her with the calculation. The visual image could be 'moves' on the number line or 'boxes' containing numbers as quantities. In both cases the visual image integrates the partitioning of numbers and grouping.



She would know that 20 groups of 3 were 60 leaving another 27 to group in 3s. 9 x 3 = 27, therefore altogether there are 29 groups of 3 in 87.

Grouping clarifies how multiplication and division are linked, giving a flexible understanding of how numbers are related in any multiplicative relation.

$$\boxed{20} + \boxed{20} + \boxed{20} = 60$$

$$\boxed{?} + \boxed{} + \boxed{} = 27$$

This interim stage of understanding division which is linked to a visual image (the number line) and a thorough understanding of the connection between division and repeated subtraction is an essential precursor to understanding more formal written methods such as the 'chunking' method:

```
87 ÷ 3          3 / 8 7
                    6 0    ( 3 x 20)
                    ----
                    2 7
                    2 7    ( 3 x 9)
                    ----
                    0 0
```

Answer 20 + 9 = 29

All teachers need to get away from the idea that a synonym for division is sharing, whether they are teaching in Year 2 or Year 9. This is clearly emphasised in the Primary Mathematics Framework and I would urge teachers who have pupils like Kerry, who are struggling with division, to use it to support pupils' learning. Perhaps then, the oft repeated mantra 'I can't do division' might begin to disappear from our classrooms.

Slough LEA

Victory at Trafalgar 21 October 1805

Leiutenant Lapontiere's 1,000 mile sea voyage from Cadiz to England, to deliver the news of the battle and the death of Admiral Lord Nelson, took just over eight days. From Falmouth Lapontiere hired a horse-driven carriage for his final 265 miles to London. He completed this cross-country stage of the journey in 38 hours. The normal scheduled stage-coach took seven days.

hfm: the Hammersmith and Fulham Council Magazine, 25 August, 2005

What is the shortest time one could travel by sea from Cadiz to Falmouth today?

Would there be a faster route? And if so, how long would it take?

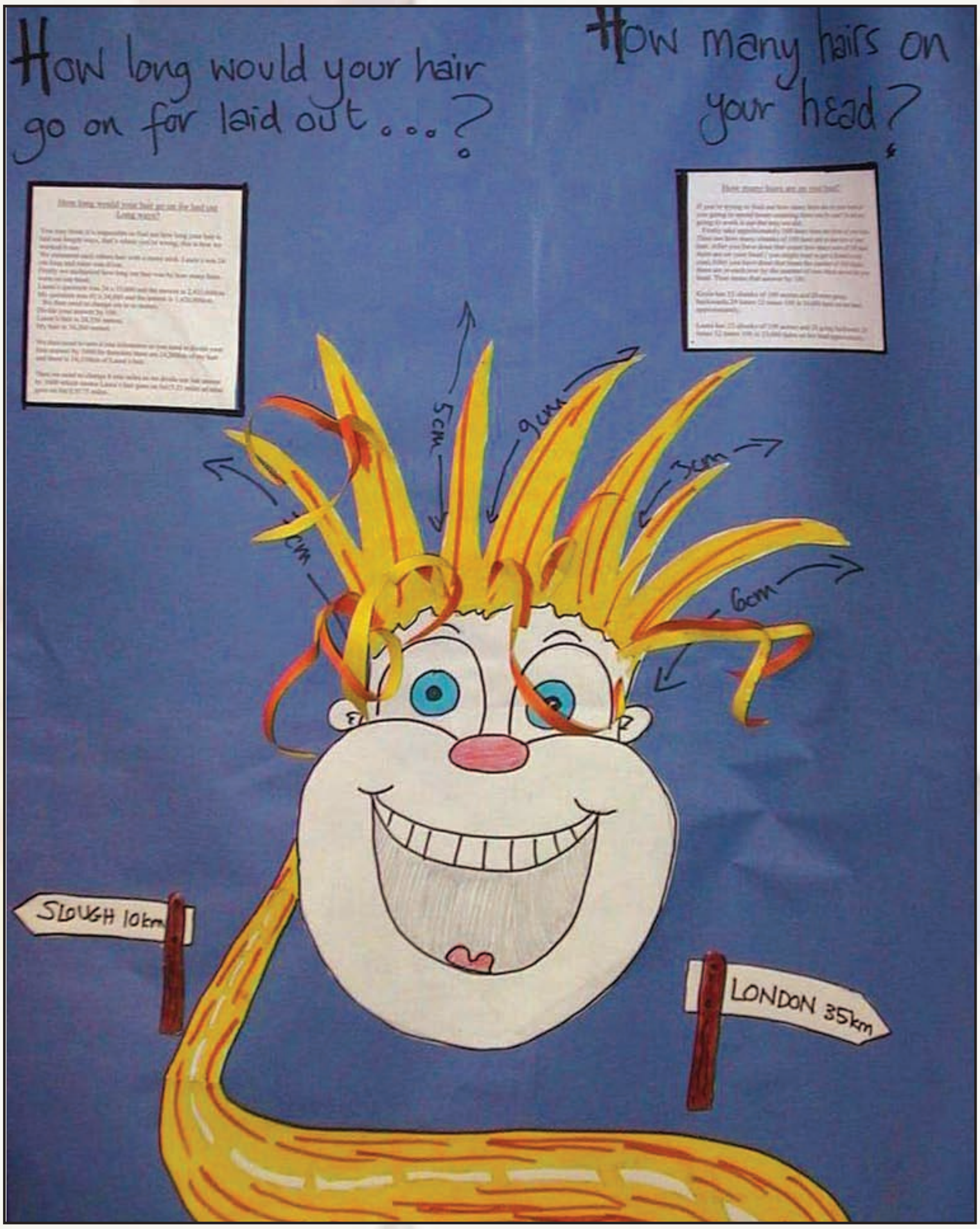
How long does the train journey take today from Falmouth to London?

What about times for the whole journey by air?

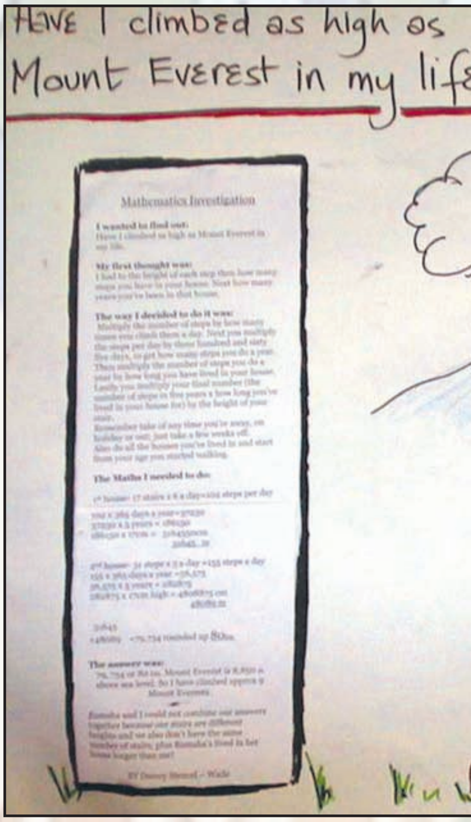
And how would the message have been sent if they had had all today's technology?

BIG numbers

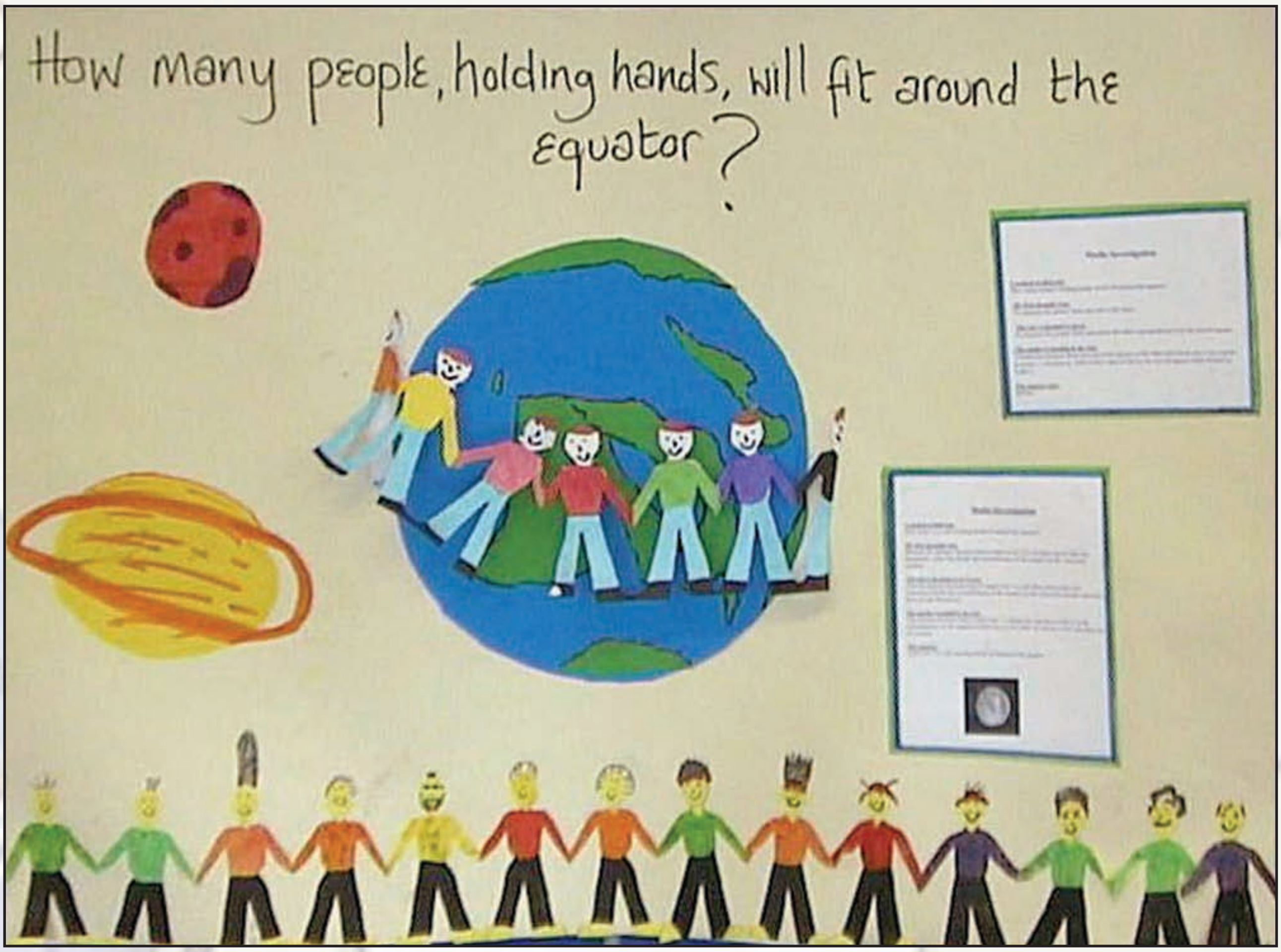
How many hairs are on your head?

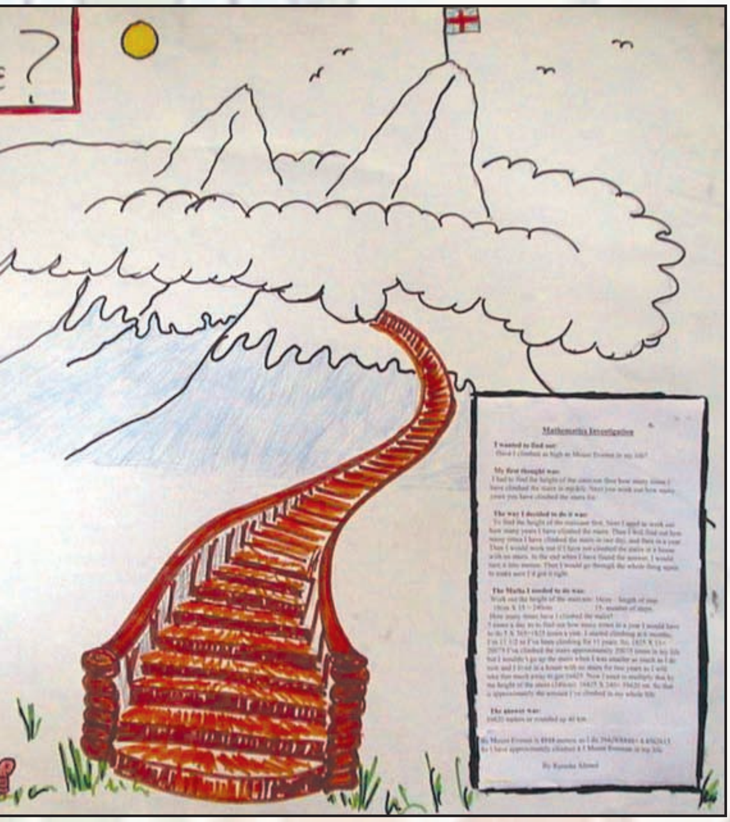


Have you climbed as high as Everest in your life?



How far would your hair stretch if all the hairs were put end to end?





How many children would it take to hold hands round the equator?

What other questions could you ask which invove big numbers?

Graham Beeden St Luke's Primary School Royal Borough of Windsor and Maidenhead, adapted from *Starting from Big Numbers* - BEAM ISBN 1 87 4099 66 2

One Teacher and a Class

Corinne Angier & Hilary Povey have attempted to describe and reflect upon Corinne's work with her 'top set' for mathematics.[1] What follows is an abridged version of their paper in which the editors hope they have captured the essence of the findings. (We believe that for any class –'top', 'bottom' or mixed achievement – the observations can usefully form a basis for improving the learning conditions of the students.)

Introduction

Hilary and Corinne analyse Corinne's and her students' reflections on how the culture of their mathematics classroom might be at odds with the needs of the participants. What students say about teaching and schooling provides an important foundation for thinking about ways of improving schools. The metaphor of spaciousness is used to evoke a culture attractive to the students. This enabled the researchers to re-investigate what it means to be a 'proper' teacher presenting spacious mathematics. Is some deviant behaviour, the authors ask, a reaction against an outmoded role being played today by some of us who are attempting to be 'proper' teachers but have not moved with the times?

The research context

Corinne's classroom contained about 30 students (ages 13 – 16 years) during their ninth, tenth and eleventh years of schooling. The information was collected from questionnaires, group interviews with Hilary, conversations with Corinne, written responses to her questions, and interviews with her in the month after they had left. Students in the main came from white working-class backgrounds in a northern town in England.

During Year 9, Corinne was given a syllabus in the form of a list of topics to be covered but she was able to work with the students in ways and using materials she deemed appropriate. In the following year she was constrained by various external pressures to implement a specific textbook-based curriculum from which, because of the stultifying effect the change seemed to be having on her students, she deviated as much as she dared. Much to her surprise, questioning the students revealed that they thought Corinne liked working with the textbook. They contrasted this with their own response whereas they were pleased with the curriculum described in this paper as *spacious* mathematics.

Spacious mathematics

Corinne and Hilary note the constraints imposed by the National Curriculum, Standard Assessment Tests and the syllabus for the school leaving examination. They describe the emphasis on procedural acquisition. In this approach teacher and textbook fracture the access so that many students think that memorising large chunks of their textbooks is the way to become mathematicians.

> *Kathryn says*, 'All the time [when] I'm doing it out of the textbooks, I'm trying to … I know I should be doing it and the answers all right so I'll write it but never mind I'll revise it when it comes … but you never get round to it.
>
> *And Frances*, ' You don't know why you're doing it sometimes.'

Spacious mathematics, however, must have room to grow as an open and creative subject not restricted to a rule-bound set of procedures.

> *James*, ' If you are learning something which is real there is a point in learning it…She makes all maths relevant…when we were doing statistics we used political stuff…to understand not only the data but the relevance it had …and how you could manipulate it... and that were much better than doing just made up statistics…it gives an understanding of the world as well as maths.'

All the students recognised Corinne's passionate enjoyment of the subject:

> *Dan*, 'She loves doing triangles!

They may have found this strange but:

> *Dean*, '…you made learning fun and enjoyable for me…your mad method of teaching is brilliant! You bring maths to life.'

Spacious teaching and learning

Spacious educational relationships allow for 'naughtiness, making mistakes and rectifying them.'

Katrina was underachieving so Corinne sat down with her.

Corinne: You remember back now to the beginning of Year 9 when you first came into my class… Do you remember how you felt…?
Katrina:…different…different people as well… when you first start working with someone you've got to learn how they work rather than how you've been taught before but…you get used to it it's all right.
Corinne: I remember you being fed up …disappointed…
Katrina: I'd got used to working [with the old teacher] …but after three years of being taught by you I got used to you as well…It's not just the teacher, it's the room as well…if you are sat in a room which is bright and lively, where you can talk…
We all felt like a family in maths. Does that make sense? Even if we weren't always sending out brotherly/sisterly vibes. We all knew how to work with each other…if like there were a new person come into the group they wouldn't know what we were like …because we were in groups we worked together…it was a big group…more like a neighbourhood with loads of different houses.

Sue enlarges the idea of a spacious learning environment:

Sue: I think when everyone is strict and sort of tense in a way it just puts you off because it gets you all psyched up…I don't think it really matters just so long as everybody's working…and I think there were always an element of people messing around but everyone … went off and did their work and there was always someone coming up with a joke which sort of made you continue because you'd had a tiny break in a way. If it's ordered the classroom then no-one really wants to work, they are just doing it because they have to and then they are not learning anything.

Connecting the metaphor

Comments were invited by Corinne after a spell of working with textbooks:

Sally: [The worst thing is] when I find it hard understanding something and everybody else seems to understand. Or when people sit there reading their answers to questions and I'm sitting there confused.

John: I might not understand something and everybody might be miles ahead… [With the textbooks there's] too much to do especially if you're doing them wrong and you think you are correct.

The use of textbooks was separating the class into isolated individuals, except for negative comparisons. Furthermore, the algorithmic style of teaching was chopping mathematics into pieces that made no sense, and the atomisation of the mathematics led to atomised social relationships. Whereas, as Neil said, 'If you are working round a table you can get…the best part of everybody…your strengths are put together.'

Epistemology

Spacious mathematics is open and creative.

For Amy: 'Maths is different from other subjects and I think it is because in subjects like history and RE you're told something is fact and you believe it…but in maths you have to prove what you are being told to do.'

Corinne allowed her students to take what made sense to them and attempted to inculcate a recognition of respect for diversity, making Katrina recognise that:

Katrina: maths is about understanding methods, sequences and getting your head around different ways of thinking. I believe maths is different for every person. Mathematics is the strangest and probably one of the most important and interesting subjects I will ever learn.

In this open and easy space students began to build a sense of their own mathematical authority:

Donna: She treats you as though you are like … not just a kid. If you say look this is wrong she will listen to you. If you challenge her she will try to see it your way.
Neil: She doesn't regard herself as higher. She's not bothered about being proven wrong. Most teachers hate being wrong … being proven wrong by students.
Frances: It's more like a discussion… you can give answers and say what you think.

Hilary and Corinne conclude that participants in the classroom need to renegotiate both the curriculum and teaching and learning relationships in ways that acknowledge the need to shift the distribution of power upon which their classrooms are predicated. They have attempted to portray one teacher and a mathematics class severing the cords of tradition that bind them, cords of institutional structures and relationships and of the mathematics curriculum. In this innovation they discern a spacious curriculum inseparably intertwined with spacious relationships allowing the growth of a more democratic mathematics.

1. Corinne Angier & Hilary Povey. "One Teacher and a Class of School Students their perception of the culture of their mathematics classroom and its construction", *Educational Review* Vol. 51, No. 2, 1999

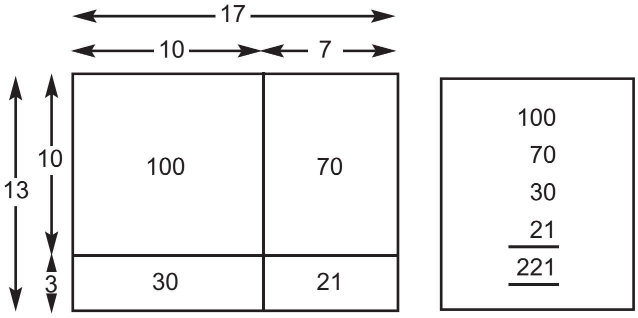
Images of division

Pupils make sense of mathematics through re-enacting, or 'picturing in the mind' actions for procedures and algorithms. Tandi Clausen-May explores such meaningful visual and action-based models linking multiplication and division.

A model for multiplication

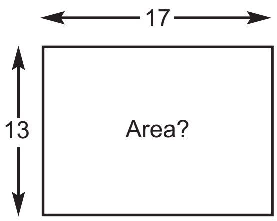
The area model for multiplication has become well established through the Numeracy Strategy and the key stage tests. It relates clearly to the standard algorithm for long multiplication, so it can help pupils who think more easily in pictures and models than in words and symbols – the 'visual learners' in the classroom – to make sense of the more formal methods.

This model can help visual thinkers to *think* about



multipication.

If we know the lengths of its two sides then we can build up a visual image of the whole rectangle. We can *imagine* the whole rectangle, even if we have not yet started to split it up into its smaller rectangles in order to work out its area.



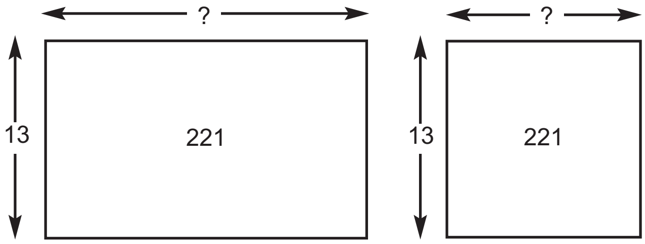
But what about division? Mathematically speaking, division is the inverse of multiplication. Since we know that $13 \times 17 = 221$, for example, we also know that $221 \div 13 = 17$. Using an area model for multiplication, we can say

> *The area of a 13 by 17 rectangle is 221.*

This can be re-phrased as a division to give

> *A rectangle with an area of 221 and one side of length 13 must have another side of length 17.*

But if all we know are the area and the length of one side, then it is much harder to visualise the rectangle. How can we tell what shape it is?



Is the rectangle this shape ?   Or this shape?
Or…. ?

The area model that serves so well for multiplication is much less helpful when we come to division. So what might we use instead?

Models for Division

Steve Chinn recommends a 'repeated subtraction' approach to division (Chinn, 2004, pg 52). This is a useful, meaningful method, but it does not lend itself well to the area-based image of a rectangle. Since we do not know what shape to make the initial rectangle, with its area of 221, it is hard to visualise what is happening as multiples of 13 are repeatedly subtracted from the total area. A more nebulous, undefined shape is needed to represent the total product that is to be divided. A pile of counters may offer a more useful 'picture in the mind' than a rectangle to think about division.

However, there is another complication when pupils – particularly visual and kinaesthetic learners – think about division. The symbolic calculation $221 \div 13 = ?$ may be interpreted in at least two different ways. It can mean, *'How many 13s are there in 221?'* Or alternatively it can mean *'What is 221 shared between 13?'* These two meanings give rise to quite different visual images. It is useful for pupils to explore both, and to recognise the relationship between them.

If we interpret the calculation to mean *How many?* then we can imagine taking heaps of 13 counters out the pile of 221 counters.

221

221 ÷ 13 = ?

On the other hand, for a *Sharing* model we start with 221 counters in the pile, and 13 empty bags.

221

221 ÷ 13 = ?

Then, following Chinn's method, we subtract convenient 'chunks' from the 221, made up of easy multiples of 13. To start with, we can subtract a 'chunk' of 130. That is, we either take ten heaps of 13 from the pile (for the *How many?* model), or we take thirteen 10s and put them into the bags (for *Sharing*).

Make **10** heaps of 13 counters...

...or put **10** into each of 13 bags.

	221
Take out **10** 13s	-130
	91

	221
Take out 13 **10**s	-130
	91

Either way, this will reduce the pile considerably. We are left with 91 counters.

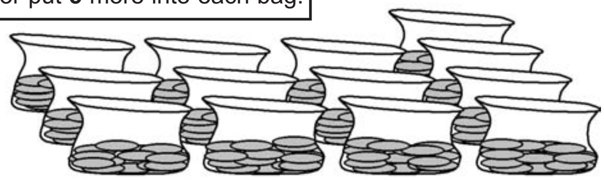
91

This is not enough for another ten heaps or another ten counters in each bag, but we can manage five more. That will remove another 65 counters from the pile.

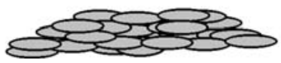
Make **5** more heaps of 13 counters...

	221
Take out **10** 13s	-130
	91
Take out **5** 13s	-65
	26

...or put **5** more into each bag.



	221
Take out 13 **10**s	-130
	91
Take out 13 **5**s	-65
	26



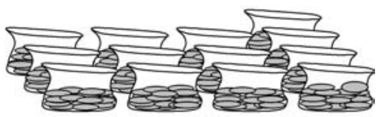
26

Now we have fifteen heaps of 13 counters, or fifteen counters in each of the 13 bags, and there are 26 left in the pile. That is just enough for 2 more heaps, or for 2 more in each bag.

Make **2** more heaps of 13 counters...

	221
Take out **10** 13s	-130
	91
Take out **5** 13s	-65
	26
Take out **2** 13s	–26
	0

...or put **2** more into each bag.



	221
Take out 13 **10**s	-130
	91
Take out 13 **5**s	-65
	26
Take out 13 **2**s	–26
	0

So now we have made 10 + 5 + 2 heaps of thirteen, or 17 heaps altogether. Alternatively we have shared out all the counters into thirteen bags, giving us 10 + 5 + 2, or 17, counters in each bag.

This example serves to illustrate Chinn's approach to division. Clearly some pupils may be able to imagine taking larger numbers of counters out of the pile at one go, while others might have to go more slowly, removing fewer counters at a time. The important point is to ensure that pupils have a 'picture in the mind' to help them to understand what is happening at each step as they divide the pile of 221 counters by 13.

Those pupils who do eventually go on to use the conventional algorithm for long division will find that the model still holds.

To divide 7956 by 34, for example, we start by finding that there are **200** 34s (6800) in 7956, with 1156 left.

$$\begin{array}{r} 234 \\ 34\overline{)7956} \\ \underline{6800} \\ 1156 \\ \underline{1020} \\ 136 \\ \underline{136} \\ 0 \end{array}$$

In the 1156 there are **30** 34s (1020), with 136 left. The remaining 136 gives us another **4** 34s, so we have a total of 200 + 30 + 4, or **234** 34s in 7956.

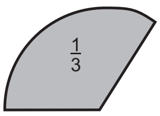
Why two models?

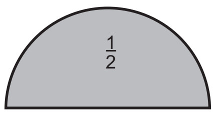
Models are not useful in themselves. They are helpful only if they enable some pupils to make sense of the mathematics. The two models for division – the *How many?* model and the *Sharing* model – are relevant in different situations.

The *Sharing* model may come more naturally as children develop an intuitive understanding of ‘fair shares’, for example when they have to share precious scarce resources – sweets, turns on the computer, pocket money, whatever it may be. This model of division links directly with multiplication by a fraction, when we find a fraction of a number or a quantity. So, for example, *Divide an apple in half* clearly means *Share it between two*. *What is three fifths of fifty-two?* asks us to share the fifty-two into five parts, and then to find the total of three of the parts.

When we think about dividing one fraction by another, however, the sharing model makes no sense at all. Take $\frac{1}{2} \div \frac{1}{3}$, for example. What could we mean by *A half shared between a third?* The idea is meaningless. On the other hand, *How many thirds are there in a half?* does mean something. Visually, it can be thought of in terms of fractions of a whole:

How many of these are there in one of these?





This model can help pupils to understand why, surprisingly, dividing one fraction by another can give a mixed number. A third is smaller than a half, so there must be at least one whole third in a half, plus a bit more. In fact, the model makes it possible to see – without any manipulation of symbols – that there is a whole third, plus another half of a third, in the half.

Images like these can help pupils to make sense of the mathematics, and so to recall and use it more effectively when they need it. Some pupils have difficulty interpreting symbols and rules. Even if they can remember a procedure for a short time they quickly forget it, and may have to ‘revise’ it again and again. Understanding mathematics takes more time and effort in the first instance, but once the algorithms have meaning they are much more likely to be memorable. More ideas and activities to help visual and kinaesthetic learners to develop ‘pictures in the mind’ on which they can base their understanding, and thus their recall, of mathematics can be found in ***Teaching Maths to Pupils with Different Learning Styles*** (Clausen-May, 2005).

Department of Assessment and Measurement, National Foundation for Educational Research,

References

Steve Chinn, 2004: ***The Trouble with Maths*** London: RoutledgeFalmer

Tandi Clausen-May, 2005: ***Teaching Maths to Pupils with Different Learning Styles*** London: Paul Chapman

Harry Hewitt Prize Winning Entry

the moray Council

Donald M Duncan
Director of Educational Services

Dear Sir

Please find enclosed my nomination for my pupil Michael Munns, date of birth 06.08.89, for the Harry Hewitt Memorial Prize.

Yours faithfully

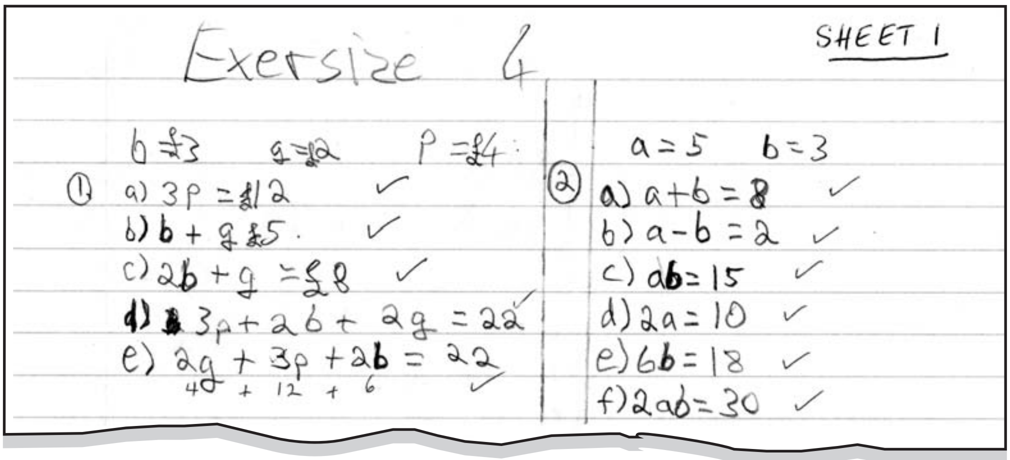
Patricia Hall

Patricia Hall
Mathematics Teacher

Michael Munns

I work at a learning Centre in Moray, for children who have problems with school, many of whom have Social Emotional and Behavioural Difficulties. Michael had been having trouble at school from other pupils and had decided to opt out. He had missed about 18 months of school, mainly the end of second year and most of third year. He started with us in August 2004. He said that he had all the Maths and English that he needed so he would not need to do it. As there were no spaces for him on the timetable at that point, this was not regarded as a problem.

At the end of September, I was helping one of the project workers with her daughters S1 Maths homework. Michael looked at what we were doing and said "Have you got any harder equations?" I invited him into my room and we found a suitable book with equations in it (Maths in Action Plus 4). He went away happily and did it, with very little trouble. (See sheet 1, exercise 4) I was very impressed, particularly with his setting out and suggested that he should use this skill to get a Standard Grade. He agreed and even agreed to doing some English as well. We settled down to complete the Foundation Course that he had started at School. This lasted 10 minutes as it soon became obvious that it was not challenging enough for him. We moved up to General level and worked at that course.


Exersize 4

SHEET 1

$b = £3 \quad g = £2 \quad p = £4$

1. a) $3p = £12$
b) $b + g$ £5.
c) $2b + g = £8$
d) $3p + 2b + 2g = 22$
e) $2g + 3p + 2b = 22$
$4 + 12 + 6$

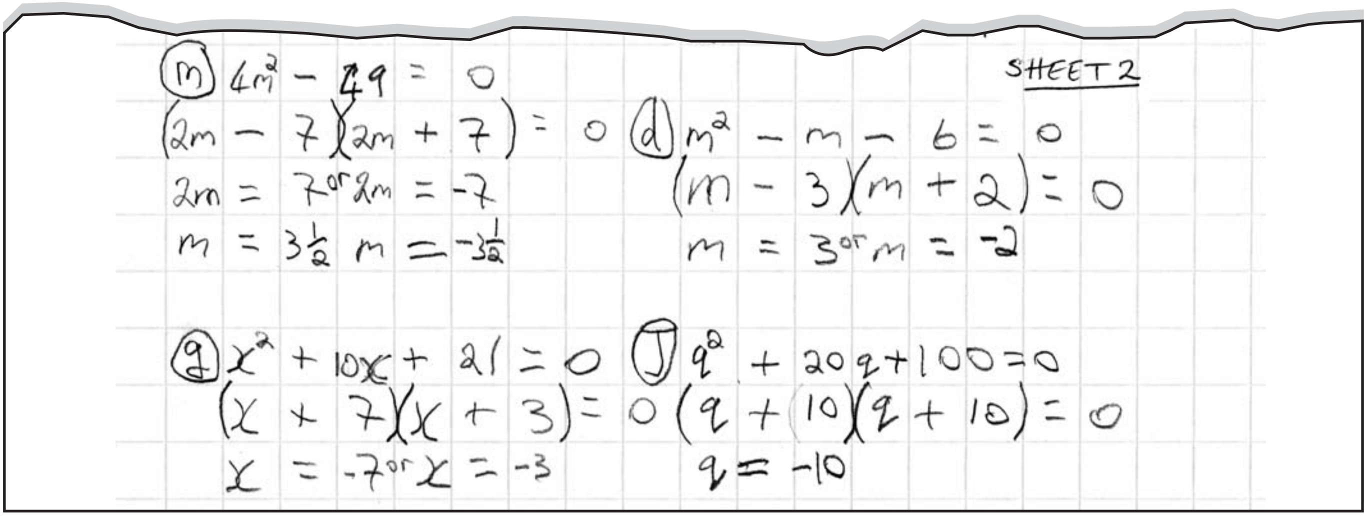
$a = 5 \quad b = 3$

2. a) $a + b = 8$
b) $a - b = 2$
c) $ab = 15$
d) $2a = 10$
e) $6b = 18$
f) $2ab = 30$

In February he sat his General Prelim exam and did so well that we started studying some selected topics from Credit level. He particularly enjoys the Algebra. He will sit his Standard Grade in May this year at General and Credit levels. Sheet 2 shows work that he completed as part of his revision.

There is a remarkable leap from the simple equations on sheet 1 and the Quadratics on sheet 2 in the space of 7 months. As we have individual tuition there is usually only one 45 minute lesson per week. Michael has been able to have 2 lessons per week with some doubling up of pupils. I am sure you can see that he has worked very hard to achieve this, against all the odds. I hope that this will put him on track for college where he can further his Mathematical Education doing Int2 and then possibly Higher.

This is why I nominate him for the Harry Hewitt Memorial Prize.


SHEET 2

m) $4m^2 - 49 = 0$
$(2m - 7)(2m + 7) = 0$
$2m = 7$ or $2m = -7$
$m = 3\frac{1}{2}$ $m = -3\frac{1}{2}$

d) $m^2 - m - 6 = 0$
$(m - 3)(m + 2) = 0$
$m = 3$ or $m = -2$

g) $x^2 + 10x + 21 = 0$
$(x + 7)(x + 3) = 0$
$x = -7$ or $x = -3$

j) $q^2 + 20q + 100 = 0$
$(q + 10)(q + 10) = 0$
$q = -10$

"School's great except for the lessons" [1]

None of the answers politicians give for truanting show any respect for the truants. Nor any admission that there may be reasons enough for them voting with their feet. **Rachel Gibbons** looks for other solutions, including where maths is inclusive of truants!

Truanting is not a new problem. I wonder how many of you, the readers of *Equals*, 'bunked off' at least once yourselves in days long ago when you were pupils who should have been in classrooms. And if you did, why did you do it? Presumably because you had found something to do which seemed to you to be more attractive than what was on offer in the classroom.

Today the press and the politicians seem never to ask whether it is something in the system that makes young people truant. Truancy seems to indicate clearly to them that something is wrong with *the truants*, or with their parents. So they only consider punishments – usually for parents - to stop it happening. As an alternative I would suggest that we might consider whether truants have special educational needs that have not even been recognised, let alone met. We should question:

- what is it about the *school curriculum* that makes truanting a more attractive option than staying in the classroom?

and then ask the second question:

- how must the curriculum be changed to accommodate the special educational needs of the truants?

But, first, a look at truanting. Patricia Stoll studied two types of truancy:[2]

> blanket truancy where a pupil is absent from school altogether(with or without parental knowledge) and marked absent in the register and
>
> post-registration truancy where pupils absent themselves from particular lessons.

Stoll found a pattern of 'high post-registration truancy'. The majority of the pupils that took part in her surveys said they truanted because they did not want to go to certain lessons, either because they disliked the subject or because they disliked the teacher.

> " Truancy is *de facto* a recoil from the curriculum which for so many pupils of 14 to 16 years of age has become dysfunctional", writes Stoll.

Certainly, the curriculum is failing the truants. So, the urgent question is: how is it failing them? Stoll found maths was near the top of the list of subjects cited by truants as being disliked. Perhaps we have to question whether the National Curriculum has suited all pupils, or whether, as Stoll suggests, it may be exacerbating truancy.

It is of course not only the content of the curriculum but the way it is presented that puts some pupils off. My memory takes me back some 30 years or more to a day when the London comprehensive in which I was teaching was festooned with strings of brightly coloured foam which had been sprayed over it by pupils - many of them in my own tutor group. Where had they got the spray cans? They had been out on a geography trip round the neighbourhood and encountered Harvey[3] who should have been with them but was on one of his frequent truancy sprees. He sold them the cans. Not for him the life of sitting at a desk reading and writing for most of the day - he wanted to join his uncle on his barrow full-time, now. Being forbidden he was practising as often as he could make the chance to do so. And the evidence in school that day seemed to be that his practice was successful. But another year in school was the life to which he was condemned. In our chat after this latest escapade, I remember pointing out that, although neither he nor I thought the school as it was then organised was really the place for him, the law said that was where he had to be for a while longer and so we had to make the best of what, in his case, was obviously a bad job. Is the current school environment any more appropriate for the Harveys of today? I don't think so. How do we make it more appropriate?

Certainly it does not seem sensible to create new institutions where we collect together all the disgruntled and not-previously-catered-for. There have been solutions that have worked in the past but on the whole they have so deviated from the traditional classroom approach that the administrators have been scared of them and all but the most adventurous teachers have given them up. Stoll comments that Kenneth Baker may have thought that the National Curriculum would be a panacea for pupils who found school 'boring and irrelevant', but it has not. Stoll considers this is because:

> It lacks sensitivity to the wishes and needs of a large number of its consumers, i.e. 11 to 16 year olds, and seeks to impose on them a curriculum with which a large number will not be able to cope.

I would like to suggest that it is useful, first, to question the short-termism of the National Curriculum with its stress on the testing of present minor skills and understanding rather than life-long learning; then, to look at some of the innovators, those teachers who have been really resourceful and changed their practice to meet the needs of the pupils in front of them. Corinne Ainger, whose work is described on page 14 is obviously one of these. The class of hers being described by Hilary Povey and herself is a top set but I would suggest that such a democratisation of the mathematics classroom as they describe could be appreciated even more by those who have probably been pushed into lower sets and are truanting because they are being treated as though "they are like … just kids." Povey and Ainger quote Ruddick et al:[4]

> … the conditions of learning that are common across secondary schools do not adequately take account of the social maturity of young people, nor of the tensions and pressures they feel as they struggle to reconcile the demands of their social and personal lives with the development of their identity as learners.

Young people need to know that their contributions are valued – and the more disillusioned they are with the system in which they find themselves, both within school and without, the more they need to feel respected. Another quote used by Ainger and Povey illustrates this:

> If one enters the educational enterprise with arrogance one's own views and knowledge quickly overpower the insights of the children. When the classroom norms are developed in such a way as to promote the exchange of student methods with mutual tolerance and respect, the children themselves become increasingly confident of their contributions and the system becomes self-reinforcing. In both peer relations and in adult-child interactions, the roles of expert, teacher, learner and novice, are flexibly drawn.[5]

Flexibly drawn roles for teacher and learner are also found in the story of E. F. O'Neill at Prestolee (see page 6) and much can be learnt about different approaches from his practice. One of the constraints he ditched was the time-table with its strict rationing and fragmentation of study in standard lesson periods ruled by the bell.

> It was difficult to break up the lessons – the sitting in rows and the formal teaching habits. … he turned the blackboards into tables. Teachers' scheme-books were asked for and collected and got lost, which put some of these people on their beam ends. The time-table, until then displayed glazed in a handsome frame, was eclipsed by a colour print of the 'Laughing Cavalier' glazed in the same frame. As a result of abolishing set lessons there were, for a time, mad rushes from one thing to another, but soon the children quietened down, finding individual interests and sticking longer at them.[6]

Maybe that is an innovation we should consider for the 21st century.

There have been other instances where young people have been keen to attend particular lessons. At North Westminster Community School in the 80s SMILE[7] gave teachers the chance to devise individual programmes of mathematics study appropriate to extremely diverse groups of students. Several members of the mathematics department reported instances of 'post-registration' attendance rather than truancy - where students sloped into school purely to come to maths lessons and no others. Certainly the present educational scene has to change if we are to make it inclusive for the truants. And it is you, the teachers in today's classrooms, who have to change it. You may not be able to replace the time-table with the Laughing Cavalier but change procedures you must. We would welcome descriptions of the successful meeting of truants' needs.

Fulham

1. Jean Ruddick quoting a child she had interviewed in a study funded by the ESRC. Found in Margaret Brown, 'Clashing Epistemologies: the Battle for control of the NATIONAL Curriculum and its Assessment', *The Institute of Mathematics and its Applications* Vol. 12. No. 3. 1993
2. Patricia Stoll, "Absent pupils who are officially present", *Education Today*, Vol 40 No 3
3. Name changed to preserve anonymity.
4. Ruddick,J., Chaplain, R. & Wallace, G. (Eds) *School Improvement :what can pupils tell us?* London: David Fulton, 1996.
5. Confrey, J., 'A theory of intellectual development: part III', *For the Learning of Mathematics* vol 15 no. 2
6. Gerard Holmes, *The Idiot Teacher: a Book about Prestolee School and its Headmaster E. F. O'Neill*, London: Faber and Faber, 1952 pp. 53, 54
7. The Inner LondonEducation Authority's Secondary Mathematics Independent Learning Experience.

Correspondence:

PENWEDDIG
Ysgol Gyfun Gymunedol Penweddig Ffordd Llanbadarn Llangawsai Aberystwyth 5Y23 30.N
T (01970) 639499 F (01970) 626641
E ymholiadau.penweddig@ceredigion.gov.uk

Dear *Equals*
I am Head of mathematics at the comprehensive above. We have about half a dozen pupils who have been diagnosed with varying degrees of Asperger's Syndrome.

During a recent discussion with a parent, I wondered whether there existed some material, website or research which gave specific tips or algorithms for teaching or helping pupils of this type. I have not found much written on the subject.

Yours Sincerely

Huw Gwyn Chambers

Mr H G Chambers
Head of Mathematics Department

Queensmill School,
Clancarty Road,
Fulham,
London, SW6 3AA
Tel: 020 7384 2330 Fax: 020 7384 2735

Dear Mr Chambers

Your letter has been passed on to me because, besides being a member of the *Equals* Team, I have been teaching children with autism for the last 8 years at Queensmill. I am the mathematics co-ordinator.

I know of no algorithms for the teaching of mathematics to children on the autistic spectrum; and as far as I am aware, Asperger's syndrome refers to children at the 'upper' end of the autistic spectrum. However, I shall offer some suggestions that work here, at least.

1. Use apparatus wherever possible to support the spoken word
2. Keep language simple and to the point.
3. Give time for language to be processed.
4. Establish routines that are workable and keep to them.
5. Give written instructions to support the spoken word.
6. Eliminate as many distractions as possible.
(I have the blinds down in my classroom all day to avoid light flickering on the paintwork.)

for further information and advice, I suggest that you contact
the National Autistic Society at
393 City Road,
London EC1V 1NG
Phone: 020 7833 2299

I hope to be publishing some materials by the start of the next school year, but these will be aimed at primary pupils.

I hope this letter is helpful to you and, with the rest of the team, I also hope that you will write up for *Equals* some of the experiments you try in the classroom.

Yours sincerely

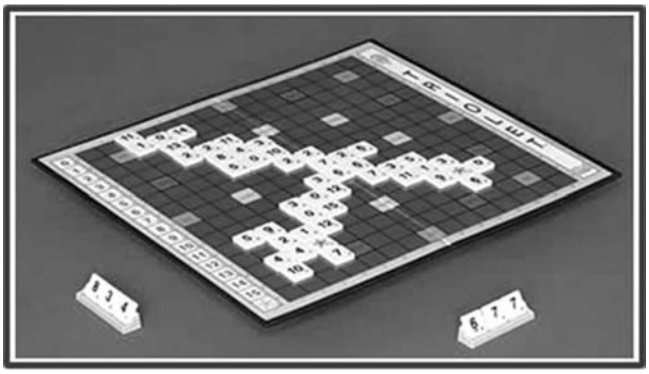
Tim Bateman

Reviews

Review
Jane Gabb

Triolet
Available through Coiledspring Games Ltd.
For 2-4 players, aged 8 and above.
£22.50 plus P&P.
P.O. Box 175
Clifden Road
Twickenham TW1 4XP
0870 446 1515
www.coiledspring.co.uk

Triolet is a Scrabble-type game using numbers instead of letters. The aim of the game is to make sets of 3 tiles which add to 15. The tiles are numbered 0 to 15 and the board is 15 x 15 squares with **double**, **triple** and **bis** squares. Players have 3 tiles on their rack and take it in turns to place one, two or three tiles on the board, joining up with those tiles already on the board.



'3 tiles side by side must equal 15'

The scoring is rather complex, being made up of several elements:

- The numbers on the tiles put down e.g. a 5 tile scores 5
- A bonus of 15 when a trio (3 tiles adding to 15) is made
- A bonus of 50 when a triolet (all 3 tiles in a player's hand adding to 15 and placed on the board) is made
- Doubling and tripling the above score if landing on a double or triple square

In addition if the tiles are placed on a **bis** square, the player can draw fresh tiles from the bag and take another go.

There are also 'joker' tiles which can be used for any tile, much like blanks in Scrabble.

I have always enjoyed Scrabble and so approached this game in a positive frame of mind. The game itself was quite fun, finding ways to make 15, and exploring different possibilities to place the tiles. However, I found the scoring very cumbersome. Scores very quickly got into the thousands and each turn was quite difficult to score. This slowed the game up, though we did get quicker as it went on.

If I were playing this game with children with learning difficulties, I would firstly simplify the scoring, leaving out the scores given by the individual tiles and just scoring for trios and triolets. The game could be further simplified by restricting the set of tiles to those up to 10 say (or any other appropriate number) and changing the target score to 10.

This is a fun way to practise addition and subtraction up to 15. You have to keep adding pairs of numbers together and subtracting them from 15 to find out what is needed, and because this is in the context of a game it doesn't feel like work!

There is also a website www.triolet.co.uk with useful hints and tips to help you play the game.

The Royal Borough of Windsor and Maidenhead

Review
Mark Pepper
Grey's Essential Miscellany
Duncan Grey March 2005
ISBN 0-8264-7491-8
£8.99 Continuum

Miscellany is an entertaining book that provides a vast range of quotations and statistics from the world of education. Its 18 chapters cover topics as diverse as absence, workload and stress, behaviour and workload and school meals. The book is a useful reference for facts. For example, it lists the names of all Secretaries of State for Education up to and including Ruth Kelly (2005).

Among many interesting tables of information is one entitled '*How do we compare with Europe?*' This lists the mean scores of 15 year-olds in reading, mathematics and science. The United Kingdom figures are impressive with positions of 3rd in reading, 2nd in mathematics and 2nd in science. It is unfortunate that the British media chose not to give headline coverage of these impressive achievements.

In another section entitled '*Challenging Oxbridge Interview Questions*' the mathematics question is: '*What would you say if I told you that you have more than the average number of legs?*' [The answer can be found at the end of this review]

Thumbnail sketches of great educationalists are given starting with William Wanflete (Bishop of Winchester 1447-1486). A chapter is devoted to educational fiction and even includes a list of all 37 members of '*The Remove*' at Billy Bunter's Greyfriars' School.

Whilst there is a comprehensive bibliography, there is no index which would have been particularly useful in a book that contains so many diverse facts. A further disappointing aspect of the book is that sources of information are not referenced.
Nevertheless, I would strongly recommend this book as a worthwhile addition to any staff library and individuals would also enjoy browsing through it.

It is true! The vast majority of people do have more than the average number of legs. This can be explained by the fact that the small minority of the population that have one or no legs marginally reduce the MEAN score of the population
(Editors: N.B. It depends what you mean by average. The MEDIAN is 2 and so is the MODE.)

Linden Lodge School