Equals

mathematics

and

special educational needs

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Letter from Darfur

- *Equals* Exclusive

Equals mathematics and special educational needs

Letters and other material for the attention of the Editorial Team to be sent to:
Rachel Gibbons, 3 Britannia Road, London SW6 2HJ

Equals e-mail address: equals@chromesw6.co.uk

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Editorial

By the time this editorial is read the result of the UK General Election will be known and the campaigning and the hours of tedious media coverage will be long forgotten. Thoughts will be turning to far away holiday venues, long lazy days on sandy beaches and many will be dreaming of a summer in which the 'flannelled fools' of England bring home the Ashes after so many barren years! We can dream!

An understanding of mathematics seems all the more important at times of general elections where politicians bombard us with statistics more often than not slanted to put spin on an issue in favour of their party. Making sense of this is very difficult but an understanding of mathematics is essential if you are going to have any chance. The use of graphical information is nearly always flagrantly misused. I am always intrigued by the claims that under certain governments there are so many new teachers, police or nurses and yet there always seems a shortage of all of these key workers. What always seems missing in the information given is how many people leave these professions! One of the more extraordinary statistics we heard at this time was the fact that if Labour and the Conservatives both achieved 34% of the vote then Labour would get a sizable overall majority. Perhaps someone could explain that one to us – that hardly seems democratic!

One of the initial thrusts of the election campaign was the commitment to improve school dinners. Jamie Oliver was seen spearheading this initiative for the government as a means to reducing the growing problem of childhood obesity. In stark contrast, the theme of world poverty has hardly been mentioned on the hustings. The terrible situation in Darfur, Sudan has been largely ignored by the media during this period. Our cover and centre spread highlight the issue and give us ideas as to how we can raise awareness of these problems with children in mathematics lessons. Surely this is what citizenship is really about not whether one or other politician will be better for us personally.

We have two articles in this issue on 'Wave 3'. An interesting title when you think of the damage that waves can do. Teachers do seem to be drowning under the paperwork of more and more waves of materials so maybe it isn't such an inaccurate term. However, these materials concentrate on developing the mathematical ability of those pupils who, up to now, have been somewhat neglected by the Strategies and to whom *Equals* is dedicated. Mary Clark introduces the format of the materials to us and Jane Gabb discusses how she has started to use them. Perhaps, the government White Paper *'Every Child Matters'* will shift the agenda from targeting a few pupils to jump through the Level 4(Year 6), Level 5(Year 9) and GCSE Grade 'C' hoops to achieve targets and genuinely concentrate on ensuring the all children regardless of ability are 'targeted' to do the best they can. Going back to the title 'Wave 3', is there someone paid to invent ridiculous titles for initiatives. At the moment we have LIL and LAL, RAPs, Ped Packs, 'Waves 1, 2 and 3' and G&T to name but a few. Why are we so prone to acronyms in education? I would like to launch the 'Campaign for the Abolition of Acronyms and Ridiculous Titles!' (CAART). Oh no, what have I done?

Visual and kinaesthetic approaches to teaching are well represented in this issue. Stewart Fowlie describes a visual approach to the teaching of multiplication and division and Mark Else talks about how he uses children as digits to develop an understanding of place value.

Equals was represented at BCME this year and a report of the session is presented. It looks at the importance of providing an appropriate diet for SEN pupils and the importance of CPD. *[Three more acronyms – sorry!]*.

Poetry has featured in previous editions of *Equals* and the reproduction of poems of Andrew Marvell and John Donne show how the poets of long ago understood the aesthetic nature of mathematics something we are in danger of losing if we become too mechanistic in our approaches to teaching.

Finally, can I remind everyone of the Harry Hewitt Prize? We are still accepting entries for this annual award which celebrates pupils overcoming difficulties in mathematics. We need to have originals of the pupil's work and a short description of how their teacher helped the pupil overcome the difficulty they had. Please send your entries to Rachel Gibbons at the address on the contents page. The pupil can be any age – we have had an adult winner of this award in the past!

Enjoy the summer!

Visual numbers : from counting to proportions, with a twist towards calculations.

Starting from diagrams of stairs and their corresponding slopes, **Stewart Fowlie** explores a visual representation of multiplication and division. In his experience dyslexics find this approach helpful (and obvious) but more able children who have good memories sometimes have never thought in this way.

All teachers are aware of the importance of starting counting with real objects. These may then be represented by counters or matches, and then by strokes written with a pencil on a sheet of paper. Children are then shown how to use their fingers to represent the objects. If they are asked to add two numbers which come to more than ten, they will start again: if they are telling you the answer, they will say the final number, and then mention the ten. So seven add six is three and ten, which they learn to call "thirteen".

If they are writing they will write 7 + 6 = 13. Notice that this is not using the concept of place value, it is merely a convenient way of indicating that you have counted all your fingers, and gone a bit further. There might be something to be said for writing X (a simplified picture of two hands) as the Romans did rather than 1. Then either X3 or 3X means the same thing, just as 7 and 6 is the same as 6 and 7.

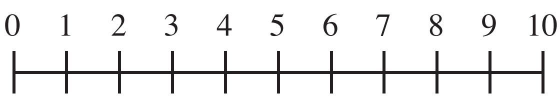
While different teachers may introduce the idea of place value differently, one way is to use 1p and 10p tokens to represent amounts of money. 56p + 37p then becomes 5 10p's, 6 1p's, 3 10p's, 7 1p's, or 8 10p's and 13 1p's, which can be changed into 9 10p's, 3 1p's, that is 91p.

The number line: using the sense of size of numbers

At the same time, children may meet the idea of arranging numbers along a line:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 .....

My recollection of this as a pupil was that it made it easier to add and subtract. For example to add 6 to 7, you pointed at 7, and then moved along one at a time counting 1,2,3,4,5,6 getting to 13. Nowadays most children meet a number line like this:



In my day this was a ruler, the numbers were an inch apart and went up to 12. Nowadays it seems to be a picture of where someone is when they have taken a certain number of steps. To begin with we just used our rulers to draw lines which were a whole number of inches long. It would have been instructive to have been asked to measure lines and say how long they were, to the nearest inch. Then if we added one line to another (|_____| + |__________| = |______|_________|) and measured the result, correct to the nearest inch of course, we might get 3 + 5 = 7 or 8 or 9.

Later on we had to draw and measure lines using the 9 divisions between each inch (a big one in the middle and 4 on each side). In those days we hadn't heard about decimals, and were told to draw lines for example $3\frac{1}{2}$ or 2 4/10 inches long, and to measure lines giving the answer similarly. Nowadays we would be asked to draw lines 3·5cm or 2·4cm long.

It should be appreciated that whereas 3 + 5 = 8 is seen by counting on, 3·5 + 2·4 = 5·9 is not. All children see the 3·5cm as if they were measuring it. The 3cm is on the left, and the 0·5cm on the right, and similarly for the 2·4cm. The answer will be 3 + 0·5 + 2 + 0·4. They need to appreciate both the commutative and associative principles to see that 3 + 0·5 + 2 + 0·4 = 3 + 2 + 0·5 + 0·4 = 5·9. There is the question of accuracy to be considered.

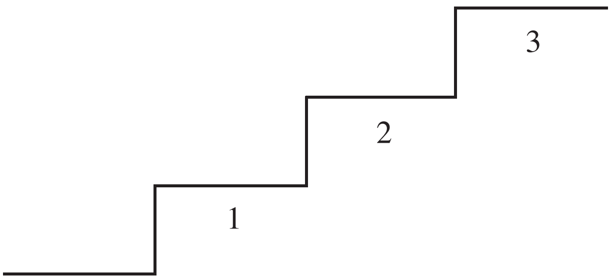
In the line above (|_____|_________|) the one thing we can be sure of is that the length of the whole line is precisely equal to the lengths of the two parts added together.

The Greeks, whose number work was limited, and who mainly worked by drawing plans, used the length of a line to represent numerical quantities. Much of Euclid's geometry was really about this. Children begin using the concept of length from birth. The following makes use of this concept, rather than counting. While we shall use numbers to name lengths not to count.

Here is a way of developing concrete images which should lead to real understanding of working with continuous quantities rather than with the counting numbers.

"Steps" and "stairs": combining numbers going across and up

Using an ordinary number line and applying it to walking is difficult in that different people take steps which are different lengths and there is nothing to see. The model of a stair avoids these difficulties.



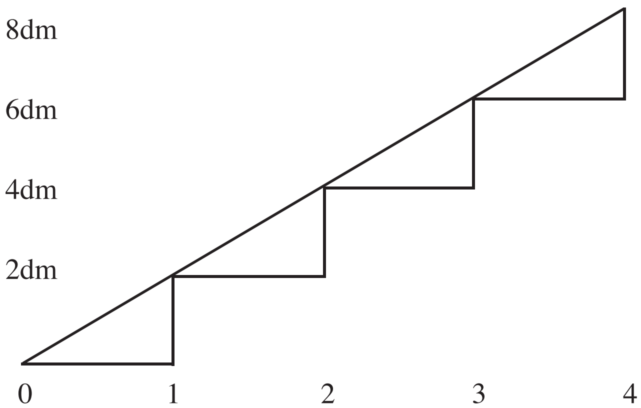
For addition and subtraction, something like this is all that is required. In due course the numbers can be omitted.

Questions are based on the following:

Tom climbs up 2 steps, and then up another 3: how many steps has he gone up altogether?

John climbs up 4 steps, and then goes down 2 steps. Where does he finish, compared with his start?

Peter is on step 2; he climbs up 3 steps. What step is he on now?

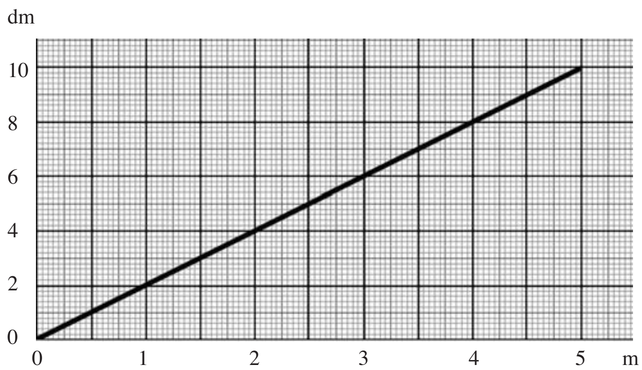


The height of each step is 2dm: how much higher is Mary when she has climbed 3 steps?

It should be easy to see how the result can be calculated, but also how it could be read off from the diagram. It is also easy to see how to read off the answer to the reverse problem, how many steps has Mary climbed when she is 6dm high?

The sloping line has been drawn to prepare for the next stage, but before doing that it would be valuable to see that a similar diagram can illustrate the cost of a number of articles costing 2p (or £2) each, or the mass of a number of articles where the mass of each is 2kg, or even just the number facts 2x1=2, 2x2=4, 2x3=6, 2x4=8 as well as the facts 2÷2=1, 4÷2=2, 6÷2=3, 8÷2=4.

Slopes: showing the decimal fractions



A ramp is built beside the steps, represented by the sloping line in the above diagram. The "steps" are still 2dm high, but are also 1m wide. In this diagram the ramp is drawn without the steps, on ordinary graph paper.

It would be better for younger children to draw it on squared paper, adding the steps if they like.

The new idea here is that the diagram shows how high someone is when they are above the middle of a step, and this gives the following results:

2 x 0·5=1, 2 x 1·5=3, 2 x 2·5=5 and so on, which can of course be written as 1÷2=0·5, 3÷2=1·5, 5÷2=2·5

It would be possible to estimate how high Mary was above, say, the point 3·4m along, illustrating that 2 x 3·4=6·8.

Notice that if the measurements across were multiplied by 10, then the measurements up would also be multiplied by 10. Further if each step was not 2dm, but say 1·3dm, the sloping line would still start at 0, but above 1 would be at 1·3. From that diagram, the results of multiplying 1·3 by any amount between 0 and 5 could be read off.

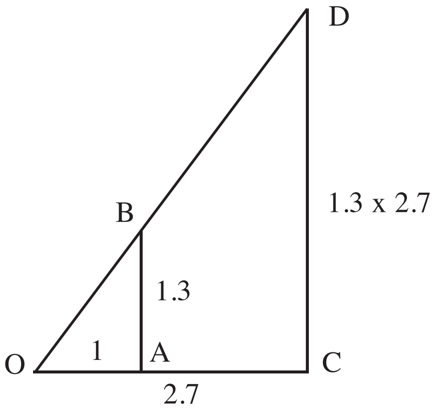
(A note which can be ignored unless you are worried multiplying a length by a length and getting a length rather than an area. This is because the 2 which appears in each number fact is really a ratio of height to length along.)

Using measurements (and embedded similar triangles) for multiplication

We usually only want to find the result of multiplying one number by another. All that is required is to draw the following diagram.

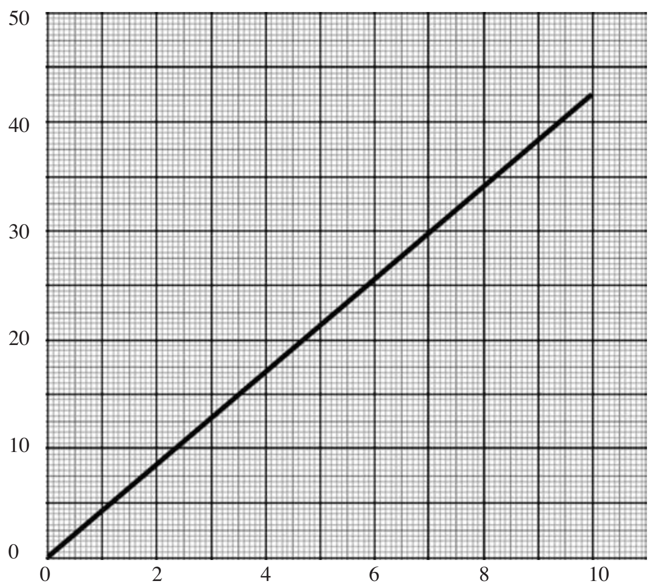
It may be drawn using only a ruler, and a second sheet of paper, though a set square makes it easier to draw the "up and down" lines, and a pair of dividers (or compasses) makes it easier to measure lines.

This diagram shows how to draw a line 1·3 x 2·7 units long:



Start by drawing a line 2·7 units long (this is OC)
Mark the point A on the line 1 unit from O.
Draw the line AB straight up through A and make it 1·3 units long.
Draw the line OB and extend it so that the line straight up through C will meet it.
Draw CD. Its length will be 1·3 x 2·7 units long.
Notice that if you interchange 1·3 and 2·7, you will get a different diagram, but should get the same length for the answer.

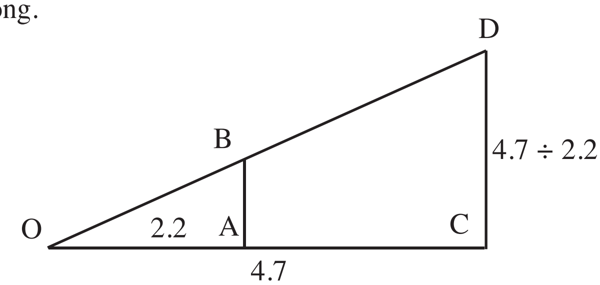
Here is a diagram on squared paper for multiplying 4·25 by any number up to 10.



Notice that instead of drawing a line 4·25 up from 1, it is easier (and more accurate) to draw a line 4·25 up from 10.

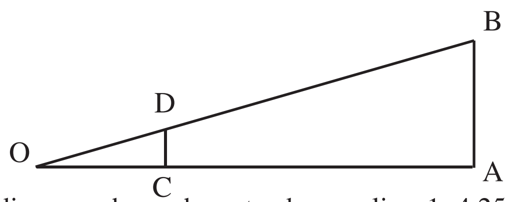
Notice that if you want to multiply 4·25 by numbers between 0 and 1, such as 0·2, 0·4,… accurately, all you need to do is to divide the 10, 20, 30… by 10.
If you want to multiply 4·25 by say 3·7, the easiest way is to find 4·25 x 3 and 4·25 x 0·7 and add the results together. (This may be related to the way that long multiplication is usually introduced)
Notice also that the results of dividing 10, 20, 30, 40, 50 by 4·25 can also be read off from the diagram.

Triangles can be used for division too!
From the examples already considered, it should be clear that points showing the answers to division of various quantities by a particular number will lie on a straight line also. Instead of the point above 1 being 4·25 up, the point above 4·25 will be 1 up. When the line joining that point to the zero point has been drawn, the point above 1 will be 1÷4·25. This shows that multiplying by 1÷4·25 is the same as dividing by 4·25, and that multiplying by 4·25 is the same as dividing by 1÷4·25. 1÷4·25 is called the reciprocal of 4·25. There is a button on most calculators labelled x^{-1} which will turn the last number entered into its reciprocal (the reciprocal of 4·25 is 0·235 approximately)
This diagram shows how to draw a line 4·7÷2·2 units long.



Start by drawing a line OC, 4·7 units long.
Mark the point A on the line 2·2 units from O.
Draw the line AB straight up through A and make it 1 unit long.

Draw the line OB and extend it so that the line straight up through C will meet it. Draw CD. Its length will be 4·7÷2·2 units long.



This diagram shows how to draw a line 1÷4·25 units long, that is the reciprocal of 4·25. You should choose as large a scale as possible.
Start by drawing the line OA 4·25 units long. Mark the point C on the line 1 unit from O.

Draw the line AB straight up through A and make it 1 unit long. Draw the line OB. Draw CD straight up through C. Its length will be 1÷4·25.

It may be noticed that this approach is not really about multiplication and division as most children see them. It is rather about growing and shrinking, and really just about growing (bigger or smaller). Should we not be seeking to enlarge understanding and enhance perception, rather than to multiply the number of different methods in the child's mind?

Many students may not fully grasp the reasons why the multiplication and division 'work' in the examples above. That is because they do rely on the idea of proportionality translated visually into similar triangles one of which has 1 as the length of one side. But all students should appreciate that, in a multiplicative relationship, quantities relate to each other in a precise way and that the numbers given in measurements will reflect those relationships.

Edinburgh

Moving numbers

Mark Else describes a simple activity that can also be incredibly powerful. It can be used as a mental and oral starter to the lesson, but by using the right questions can be extended to give pupils valuable insights into the workings of the place value system, including very big numbers and how to write them, and very small numbers. It can also become a useful teaching tool for showing how numbers behave when they are multiplied (or divided) by powers of 10. As a lesson, it should suit all learning styles (visual, audio and kinaesthetic) allowing all pupils to access it and get something from it.

I teach in a secondary school, working primarily with pupils at the lower end of the ability range (generally pupils working at levels 2-4 in KS3). One of the biggest problems that some of these pupils face is that they just 'don't get' maths despite going over and over it again all the way through their school career so some of them are switched off. My task therefore is to find new and different ways of presenting the subject to them to help them understand. Many of them don't really like written work either so anything where they can discuss or be active in the lesson seems to suit them better.

Outline of activity

Setting the activity up is very simple, all it requires is a number of A4 cards each one with a digit from 0 to 9 on it and an extra card with a decimal point on it (I use a different colour here to emphasise that it behaves differently from the numbers). It may also be useful to have some extra 0s standing by for when you move into decimals or multiplying. When working with smaller groups, I will often give each pupil a card of their own as they will all take part at some point during the activity whereas with bigger groups I will give cards out as they are needed. I will then either ask a group of pupils by name to come and stand at the front to make a number (e.g. Luke, David and Sam make a number) or ask the group to make a specific number (who has the cards to make the number 263?).
The activity can then go in any direction you want depending on the questions you ask.

Basic place value

I will usually start off by asking someone to read out the number that has been made. This is particularly useful as the number gets bigger, or if zeros are involved in the number. It may take several attempts to get the number right. This is then followed up by a series of questions about the number at the front:

e.g. What does the 3 stand for/which column is the 3 in?
What does Luke stand for/which column is Luke in?
Who represents hundreds?

Once the group get the idea with a small number, I will start to increase the size of the number into thousands and above. The obvious way to do this, particularly with large classes, is to use a different set of pupils so that more can become involved. One way that I find useful however is to simply get another pupil to stand at the front of the number so that 263 becomes 5 263 and so on. This helps with the reading out of the numbers as pupils can build on what they already know.

To consolidate this knowledge of place value, I then get the pupils to create a new number with the same cards by asking them to swap number cards and stand in their original position.

I then go through some of the original questions:

What number do we have now?
What does the 3 stand for?
What does Luke stand for/Which column is he in?

This last question is of course a slight trick question as Luke will be in exactly the same place value column as before. This line of questioning will allow you to explore the idea that numbers can mean different things when they are in different places in the number (i.e. are held by different people)

An alternative to this is to allow everyone to keep their number and change places so that they stand in a different position to create a new number. The questions can be repeated as above but now the emphasis is on pupils behaving as the number instead of behaving as the position in the place value system. It may depend on how you want to extend the activity as to which of these options you follow.

Finally, the introduction of a zero into numbers can generate further useful discussion:

What does it stand for? What does it mean/show?
How do we read out numbers with zeros in them (e.g. 50 263, 3 028)?
Why do we need it there? What is the point of it?

Extending into big numbers

Having used and developed this teaching resource over the last couple of years, I have found that the most difficult area for the pupils has been distinguishing between ten thousands and hundred thousands, particularly when reading out numbers. It is worth spending a little time on this to ensure that pupils understand. Recently however I have started extending this activity to explore millions and even billions. In doing this, the pupils inevitably find it more difficult to read the numbers as more digits are added and they get confused as to which digit means what. This has led to some useful discussions on why it is difficult and how we could make it easier for ourselves. Through this discussion, I have been able to introduce the idea of having gaps every three numbers (including agreeing that we must start from the right) and also explore the patterns within each group of three (effectively a units, tens, hundreds repeating pattern). This enables the pupils to concentrate on reading three numbers at a time and then add the appropriate ending (million, thousand etc).

This activity inevitably leads to questions about billions – what is a billion? (Remember, there is the old UK definition of a million million and the now widely accepted US definition of a thousand million). Questions are also asked about numbers bigger than billions and what they are called – so you may want to be prepared!

Extending into decimals

As well as looking at large numbers, this activity can also be used to look at small numbers and decimals. When I introduce the decimal point, I impose a rule that the decimal point is 'super-glued' into position and cannot move. Any swapping (of cards or position) must therefore take place around it. This rule is more important when we use this method to multiply by 10 (see below) but is worth emphasising as soon as it is introduced to the activity.

The introduction of decimals also leads to discussions about how we read them, reading each digit separately instead of in groups (e.g. 0.14 is read as zero point one four, and not point fourteen).

Having established these two facts, the activity can proceed as before with similar questions being asked and the number becoming more complicated by adding more pupils either side of the decimal point as they become more confident. With some pupils, it can take some time to establish where the units and tenths actually are so it is worth spending a little time focussing on this.

Finally, as with the basic activity, there is plenty of scope for a discussion about the role of zero, in addition to discussion points raised above, the important point of when the zero is needed and when it isn't can be discussed, along with the idea that 0.1 is the same as 0.10. I usually tell the pupils with zero cards that they have a special job as this helps to emphasise the 'special' role of zero as a place holder or place filler.

Multiplying and dividing by powers of 10

The activities and questions outlined above are extremely useful for discussions based around place value, and provide useful starters for lessons on place value, big numbers or decimals but for me the real power of the activity comes in its use as a tool for multiplying and dividing by powers of 10. It provides an opportunity to get pupils out of the 'just add a zero' mentality and also shows how numbers move around the decimal point (rather than the decimal point moving). In addition it provides the pupils with a visual and kinaesthetic representation that helps to remove some of the abstractness from this part of maths and enables them to understand it better.

The activity can be set up as before with a group of pupils standing at the front to make a whole number. Questions can be asked about place value to reinforce or remind them of previous lessons. I then ask what would happen to the number if we tried to multiply it by 10. Answers often include 'add a zero', which leads into a discussion of where the zero has to go and the solution is that it has to go in the units column. This means that whoever is in the units column has to move to make room for them, and they have to move into the tens column, causing the person who is there to have to move up again and so on. The idea being that every number has to move up a column in the place value system (pupils move with their numbers so that they are behaving as the number would do). One of the pupils sitting down can be in charge of telling the people at the front what to do. ("Luke has to move to that position…"). The idea can be reinforced with further questions:

What does David stand for now? What column is he in?
What did David stand for in the previous number?
Who represents hundreds now?
What did they represent before?

Once again, there is an opportunity here for a discussion to introduce or reinforce the role of the zero. The pupils with a zero have a 'special' job as they are getting up and down more often as needed.

The activity can then be repeated and reinforced by using different numbers or by multiplying by 10 again. Then it can be extended to see how numbers are affected when we multiply by 100 or 1000, or if we try to divide by 10 (the numbers all move down a place and the zero is no longer needed so has to sit down again.)

Many pupils who can handle multiplying whole numbers by 10 get into trouble once the numbers become decimals, particularly if they have a 'just add a zero' mentally. Once pupils are used to the idea of digits moving up and down the place value columns, it is a natural extension to use this with decimal numbers. The point about super-gluing the decimal point into place so that it cannot be moved may need re-emphasising here so that pupils become used to the idea of the digits moving around the decimal point to the other side. Once again pupils are moving with their numbers to create the new ones and pupils sitting down are telling them what to do. As the pupils become more confident, numbers with more decimal places can be used, and numbers can be multiplied and divided by 100's, 1000's or even bigger numbers if the group can handle it. Once again, there is plenty of scope for a discussion into the role of the zero, particularly in decimal numbers as you can create numbers with leading zeros (e.g. 0.0043).

The emphasis throughout the activity is on numbers moving around a decimal point, which is glued in position. Additional points about the numbers moving together and staying in the same order can also be made. Asking regular questions about place value can reinforce the whole activity.

Extending it into written work

As a mental and oral skill, this activity is very powerful, but it is essential that the pupils also have a way to translate this into written work.

Depending on their ability, I encourage pupils to set out their sums using a table in their books as follows

	Th	H	T	U	.	t	h	
		2	4	6				x10
=	2	4	6	0				
				3	.	6	5	x10
=			3	6	.	5		
			1	8	.	7		x100
=	1	8	7	0				

Arrows can be used to show the numbers moving up a column (or more) and writing answers directly under each other and lining up the decimal points where relevant helps to emphasise this point. As pupils become more confident with this work, they find they don't need the arrows and can eventually do away with the need for a table, possibly using squares in their book to line up the numbers instead.

One important feature of any activity however is obviously whether the pupils cannot only learn from it, but whether they can retain the information in their long-term memory. During lessons, I regularly revisit the ideas of multiplying and dividing by powers of 10 through the use of a set of flashcards for short periods of time. I have found that some have easily retained the information while others need a gentle reminder of the form "do you remember the activity when…". This has enabled them to remember the skills for themselves without me having to actually tell them what to do. Constant reinforcement of this type has enabled more pupils to retain the concepts involved in their long-term memory.

Postscript

Having used and developed these ideas over a couple of years, I figured that I had the activity sorted, including all the classroom management issues, but as usual in this profession, the pupils come along and cause a problem that I hadn't thought of. I had used the activity with several groups by having them standing at the front of the class. Suddenly one of my bottom sets that usually responds well to different activities found it very difficult to focus on the lesson and to stand still with the cards at the front. Messing around with my whiteboard seemed to be a particular attraction! I got around this by setting out a row of seats so each pupil in the number had a seat and each seat stood for a different column in the place value system. The person representing the decimal point became super-glued to their seat and everyone else moved around him. This method became particularly useful for working out which spaces needed filling with zeros and which didn't, and for working out how many spaces to move when you multiply by 10, 100, 100 etc. It was effective because the group was small and we could rearrange the classroom and all sit around the front so that everyone could see properly and it also solved the problem of my fidgety group.

Correspondence:

Where are your letters? Some of you who wrote about 'Gypsy Maths' were too late to be included in Vol. 11 No. 1 and the editors wish to thank you for your letters. We believe that every reader must have at least one idea that is worth sharing with others, so please write about your most exciting and effective invitations to mathematics which have appealed to those who appear to be reluctant learners.

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Letter from Darfur – *Equals* Exclusive

In a letter to a member of the editorial team **Helen Austin** describes something of her work for Médecin Sans Frontières in Darfur.

I'm still in North Darfur, high in the Jebel Marra (mountain) range. Jebel Marra used to be the garden of Sudan. It 's fertile, green, with rich volcanic soil. Our camp is very basic: tents and plastic sheeting (a bit like a mini refugee camp) but it's great for me as I love the outdoor life and the mountain setting is truly stunning. It has been very, very cold – especially at night - sometimes 0 degrees but it is getting much warmer now. Fortunately we have these fantastic mountaineering sleeping bags so you can sleep outside comfortably, with a priceless view of the stars. Everything grows here and most people are farmers, usually a mixture of arable and pastoral. Before the war and displacement the diet of a child would be quite different from now. The people cannot go back to farm their land and rely almost completely on food aid. ...

We've vaccinated 6,000 children against measles…Also given 18,000 children (under 5 years old, at risk of malnutrition) special food rations and non-food items such as plastic sheeting, jerry cans, blankets, etc. during two huge distributions. We have a clinic here at Kaguro (approx 600 consultations per week), an outreach programme: mobile clinics and training of community healthworkers – approx 1,300 consultations per week… a therapeutic feeding centre for the severely malnourished, currently 62 patients. On the 9th and 10th we had conducted the final food distribution. 10,000 children under 5 years received a special ration for one month, polio vaccination and vitamin A. Moreover, we screen the children for malnutrition at each distribution and the statistics are great. Our intervention has significantly reduced malnutrition in our target area. Overall the nutritional status of the children has improved with each distribution. We are very happy as you can imagine!!
We screen with MUAC [mid-upper-arm circumference].
If they fall in the red bracket this is severe malnutrition:
orange - moderate malnutrition,
yellow - at risk
green - healthy.[1]

	February	March	April
Red	0.4%	0.34%	0.13%
Orange	14%	11.2%	4.3%
Yellow	46%	31.2%	21.4%
Green	39%	57.1%	72.9%

A typical day for a child would start with a thin porridge made from the grain sorghum or the Corn/Soya blend which is given in the food distributions - no sugar added and very bland for a European child's tastebuds. Not a rice crispy in sight. The main meal of the day is at lunchtime, they call it Fatour which actually means breakfast. The adults don't usually eat in the morning. Fatour is almost always 'aseeda'. The people love it here and there is great competition to make the best. Aseeda is made from ground sorghum and it's cooked with water until it forms a thick, stodgy gloop. Once it cools it sets to a wobble. It looks like mashed potato but tastes a bit like wallpaper paste. Its usually eaten with a thin soup of lentils and if a child is very lucky, a meaty gravy. Aseeda is the staple food here. Another important dish is made with a bean which looks like a red broad bean. Quite good but always served in a bland fashion, cooked in a thin watery sauce. Peanut paste is sometimes used in sauces and occasionally some chilli. Bread is also commonly eaten, small round, flat loaves which are wonderful fresh baked.
A child rarely eats meat but this is recent and due to the war, animals looted etc., lack of means for parents to earn a living so little income to buy food. There is rarely fruit or vegetables as the people simply cannot plant due to the insecurity. A treat for a child would be rice with sugar, an orange, dates or peanuts. You often see the children scavenging underneath the trees for a small fruit. I don't know the name but it's about the size of a Kumquat, furry and has flesh like an unripe peach. In side are many seeds so you don't get much fruit for your efforts!
The portions are difficult to judge as everyone eats from the same dish but they are very modest by our standards. A couple of dishes are laid out and the family sit in a circle dipping into each bowl. They sit on the floor of course, with no cutlery or plates. I've shared several meals with families and it's a very social and pleasant way to eat. The general food rations given by ICRC include Sorghum, CSB, lentils or pulses, oil, and salt. A typical family can rarely afford to supplement this with meat of fresh vegetables.
The children in our immediate area are lucky enough to have a school. They take porridge or aseeda with them as a 'packed lunch' and always carry water.

1. See the centre spread in *Equals* Vol. 8, No.1, Spring 2002 for the Oxfam measuring tape. This tape has red and green sections with the 13th centimetre in between left white.

Our Base Compound

Comparisons

In this country there has recently been much publicity on what the younger generation eats, if not how much. Jamie Oliver's TV series has prompted many people to look at the contents of school dinners. How do your eating habits compare with those of the children of Darfur? A class study will be useful to make the comparison.

The first report of The London Mayor's *Living Wage Unit* found that one in seven London workers are living below poverty levels so what proportion of London children are living below poverty levels and are any of them suffering from malnutrition?

Class study

For one week keep a record everyday of all you eat and - with the aid of the figures in the table below table below and discussion with your teacher and classmates - work out and list in the blank table your intake of protein, fat, carbohydrate, etc.[1] (There may be other foods not listed for which you will have to find out the content of all these nutrients per 100 grams.)

Collect the results for the whole class and present your findings. You may want to find the average food intake per day, the range of intake from greatest to least for each column or you may want to construct some sort of graph to display your findings. This will have to be a decision of the class as a whole.

When you have done this you can start to consider the daily food intake of the Darfur children. This will take a bit of thought and careful estimation to define a 'portion' of each type of food. (If you think you need more information from Helen about this just e-mail *Equals* and we will contact her for you.) Having made these decisions, you can then analyse the food intake of the Darfur children as you did your own, and then write a report of your findings.

1. Table taken from John Yudkin, *The Slimming Business*, London: Penguin Books, 1962

CONSTITUENTS IN 100 GRAMS OF FOOD

Food	Protein grams	Fat grams	Carbo-hydrate grams	Calories	Calcium mg.	Iron mg.	Vitamin A I.U.	Vitamin B_1 mg.	Ribo-flavin mg.	Nicotin-amide mg.	Vitamin C mg.
Milk	3.5	3.9	4.9	69	118	0.07	160	0.04	0.17	0.1	1
Cheese, Cheddar	23.9	32.3	1.7	393	873	0.57	1740	0.04	0.50	0.2	0
Beef, steak	19.3	13.0	0	194	11	2.9	0	0.12	0.15	5.2	0
Fish, haddock	17.2	0.2	0	71	19	0.9	0	0.01	0.12	1.4	0
Eggs	12.9	11.5	0.7	158	54	2.7	1140	1.12	0.34	0.1	0
Apple	0.3	0.4	14.9	64	6	0.3	90	0.04	0.02	0.2	5
Cabbage	1.4	0.2	5.3	29	46	0.5	80	0.07	0.06	0.3	52

Teachers' notes

This project, based on information fresh from Darfur and exclusive to *Equals*, clearly will not fit into the three-part-lesson format. Nor will it be easy to set detailed lesson objectives of the "you will learn" type for a single lesson, or even for the end of the project, because pupils may concentrate on very different aspects and the mathematics is merely a means to a variety of other ends. Nor can one predict the mathematical content with any certainty: the statistics employed will vary from class to class and probably from pupil to pupil. (Questions about accuracy of measurement will arise throughout.) However, it may give, to use Ainger and Povey's term, some *spaciousness* to the classroom culture.[1] Ainger and Povey write of 'spacious mathematics' arising in activities that are mathematically rich and more likely to generate 'the temporal and intellectual space' within which students and teachers can make links both within mathematics and between mathematics and other experiences. They extend the metaphor to spacious educational relationships large enough to include the 'space to be naughty, to make mistakes and to recognise and rectify them, to move one's elbows.' This is surely a more natural way of working than the more usual lockstep: heads down, do as much of the exercise in the textbook as you can. More information will have to be sought before the pupils can get started in analysing their food intake for nutrients as only a brief list of foods is given in the table. Pupils can be encouraged to take their questions to experts in other disciplines or to consult reference books in the library.

If you have the Oxfam tapes from *Equals* 8.1 the above information could be a lead in to mid-upper-arm measurements for young children or various other age groups, considering where the 'malnutrition' point on the tape might be for older age groups.
Further projects could include:

Child poverty in London: how is this linked with malnutrition?
Are the figures for other large centres of Britain similar?

The data necessary to give an answer to any of these questions will have to be gathered by the class from newspapers, council offices, reports from government departments, reference books,etc.

1. Corinne Angier & Hilary Povey, "One Teacher and a Class of School Students: their perception of the culture of their mathematics classroom and its construction", Educational Review, Vol. 51, No 2, 1999. We shall be printing an abridged of this paper in the next issue of *Equals*

FOOD DIARY FOR ONE WEEK											
Date	Protein grams	Fat grams	Carbo-hydrate grams	Calories	Calcium mg.	Iron mg.	Vitamin A I.U.	Vitamin B_1 mg.	Ribo-flavin mg.	Nicotin-amide mg.	Vitamin C mg.

Prize

***Equals* would be interested in printing the best reports presented (before the end of July) by any class and will offer a prize from the Harry Hewitt Memorial Fund for what we judge to be the best report.**

MATHEMATICAL ASSOCIATION

supporting mathematics in education

The Mathematical Association is pleased to offer:

Arithmasticks from Eduplay UK Ltd

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Wave 3 Teaching Assistant Training

As well as providing materials for pupils with substantial difficulties the mathematics team are providing training for Teaching Assistants who will be working with these pupils. **Jane Gabb** describes a programme for a four-session course

Over the last (financial) year and in the coming funding round, local education authorities have had a standards fund grant for Wave 3 projects in mathematics, aimed at supporting those pupils who have substantial difficulties with mathematics.
To put this in context:

> Wave 1 is the daily mathematics lesson, available to all
> Wave 2 covers catch-up programmes like Springboard and booster (for pupils who might reach expected levels with some intervention)
> Wave 3 is for pupils for whom catch-up programmes are not appropriate (i.e. they are not expected to reach age-appropriate levels by the end of key stage 2.)

As part of our Wave 3 mathematics programme I recently worked collaboratively with a colleague from our Learning and Cognition team to plan and deliver 4 sessions of training for teaching assistants who were supporting with pupils working at or below level 1.

The first session covered pre-counting and early counting skills and some of the suggestions and discussion points are given below. Towards the end of this session we asked the TAs to write about any pupils they were working with who had difficulties which they were finding it challenging to address. Some of their questions and the subsequent suggestions also appear below.

The aims of this first session were:

- To understand the skills involved in counting, including pre-counting skills
- To discuss activities which support pupils who are at the pre-counting stage
- To share ideas about how work with numbers could be developed

We began by looking at the skills which underpin counting by asking the question:
What needs to be learned so that a child can count?

- Colour recognition
- Sorting
- Matching
- Classifying
- Comparing
- Ordering
- Seeing and making patterns

Some of these were then explored in some detail.

Sorting and classifying, including colour recognition:
Children need to become aware of and to be able to distinguish between:

- Colours
- Shapes
- Sizes
- Materials - wood, metal, paper
- States - solids, liquids

It's important to move beyond colour and shape into other attributes. Many commercially available sorting products seem to concentrate only on these. You can use natural objects and everyday objects to learn about different materials. It's also important that other senses as well as sight are used e.g. touch.

Participants were invited to look at a selection of commercially available materials – teddies, frogs, creatures, boxes, cups – and to discuss what questions they could ask when children are playing with these materials. They were encouraged to move beyond 'What colour is this?' to more open-ended questions like: 'Which would you put together and why?' You learn more about the child's understanding from the second question which in turn will help in knowing how to support their next steps in learning.

Matching and comparing

- Identifying the sameness of everyday things
- Visual
- Sound
- Feel
- Smell
- Taste

Participants discussed how to involve the other senses than the visual.

Sound - listen to sounds and identify them. Repeat a simple clapping rhythm. This is more difficult than visual matching – can you think why?

Feel - without sight - in a bag - try to find pairs of things which feel the same - what could you put in the bag?

Smell - use cotton wool for liquids in a lidded jar or small plastic bottle. Have pairs of the same smells - e.g. vinegar, tea, coffee.

Or herbs like rosemary and lavender. This is also difficult – do you know why?

Taste - blind tasting - start off with familiar foods - guess what it is. Then move onto the unfamiliar - is this the same or different?

With sound, smell and taste it's more difficult than seeing and feeling because you can't hear, smell or taste 2 things at the same time – you need to carry the memory of the sensation - but it's worth persevering with - children will get better with practice. And they will enjoy it!!

Seeing and making patterns

- Describe a pattern
- Make up their own pattern
- Copy or extend a pattern
- More complex patterns

Wrapping paper and wallpaper are good sources of repeating patterns. Paint and vegetable stamps can be used for children to make their own patterns by printing.

Multilink or unifix cubes, beads and plastic teddies can be used to make repeating patterns which can be copied and extended. Changing the colour or the size can be used to make the pattern.

A simple pattern using 2 colours might be: red blue red blue red blue

A more complex pattern could be: red blue blue red blue blue red blue blue

Or red red blue blue red red blue blue red red blue blue

You can also making patterns using natural objects like shells or pine cones. Invite children to bring things in (or make up your own collections):

- leaves
- postcards
- shells
- suggestions from them

Use the collections for sorting, comparing and pattern making with the objects or using them to print (e.g. leaves). There are lots of opportunities in this to develop vocabulary: 'What is the same about these?' 'And what is different?'

Table laying produces patterns which are also functional.
Don't forget clapping, singing and humming to make sound patterns.

And so to counting!
We need to be very clear what we mean by being able to count; it's a skill involving a progression and a number of sub-skills. What does it mean when a parent brings a child to school or nursery and says "My child can count to 20"? We need to assess what they can actually do - it may just be rote counting without one-to-one correspondence and understanding about what the numbers represent. A child can be said to be able to count to 20, when they can reliably count out (say) 18 bricks, if asked to, and know that whatever order they are counted in there will still be 18 bricks there. (This also involves the understanding of conservation of number.)

Some of the skills which underpin counting are:

- Numbers as labels – on houses, buses, cars, TV channels, microwave,- children begin to recognise them and learn their names.
- Saying the number names in the correct order - rote counting. This needs to be learnt and practised on a daily basis. But it is no use without the practice in counting objects - relating each number to a movement.
- Matching each number with an object – 1-1 correspondence
- Knowing that the last number counted is the number of objects in the set - they need to understand the significance of the last number; this can be reinforced with language - "So there are 3 bricks here".
- Knowing that the number will be the same if the same objects are counted in a different order. They need experience of shuffling objects and re-counting - what seems obvious to us, has to be learnt.
- Understanding that the number doesn't depend on what is counted – the 3-ness of 3. They need to count different things – say for 3 – so that 3 becomes a concept which is not dependent on cars or teddies, but is something in its own right.

Participants had various items of equipment on their tables and they were invited to look at them and discuss which of the above skills each piece of equipment could support.
The items of equipment were:

- Number cards
- Books, both big class books and small individual books

- Bead string/abacus
- Large number squares for the floor
- Skittles
- Dice with dots and dice with numbers
- Frogs/teddies

We then went on to discuss songs and stories, the use of the environment both inside the classroom and outside in the playground, and everyday routines like dressing and eating.

These were some of the difficulties which our teaching assistants identified. The suggestions came from other participants and the trainers.

Question/Problem	Suggestions
• This child is 7. He writes numbers 5, 7, and 3 the opposite way round. • Age 6. Does not form his numbers correctly, he writes them back to front.	This is a very common difficulty. Try using numbers made out of a textured material e.g. sandpaper and showing the child how to trace over the number with his finger. Magnetic numbers are also useful for this. Take one number at a time – do some tracing with the finger, then use paint on the finger to make the same number on paper, before using large crayons and then a pencil.
• Age 5. Can recognise numbers 0-10 can order numbers 0-10 but cannot write numbers 0-10.	Experiences as above would be appropriate. Also making numbers out of playdough or pastry.
• Age 7. Does not recognise numbers, has to go to back counting from 1 to remember how it looks. • Age 5. Has trouble identifying numbers if not in a particular order.	Use matching activities – number cards to match to pictures of say 4 cars, 3 trees etc. Use just a few numbers to start with (say 1-4, fewer if even this is difficult) and gradually build up. The above activities involved with tracing, drawing and making the numbers would also be useful here.
• Difficulty matching number with object. • A 5-year-old child tends to miscount when counting objects, even when encouraged to point to each object and count slowly. • Problem with matching 1 to 1. He can't count accurately. If he had beads or cubes, he wouldn't count them out accurately to a given number.	When counting small sets (say 2 or 3 objects to start with), make a large movement for each number as the object is counted, moving it from one place to another. Encourage a rhythm with the spoken number and the movement. Count steps taken as well as objects. Use any of the above suggestions as well.

Royal Borough of Windsor and Maidenhead

Pages from the Past

The Poetry of Geometry – or the Geometry of Poetry

At the time of writing the *TES* has published an article entitled "Tables kill off love of reading." These are not multiplication tables (which may well, if treated wrongly, kill off the love of number) but league tables. David Bell has complained, we are told, that the beauty of the language of poetry is being lost because poems are being used as literacy manuals. In the same article Professor Celia Hoyles is likewise complaining that league tables and targets hinder pupils' enjoyment of mathematics.

It seemed worthwhile to remind ourselves of some of the beauty of the language and of the metaphors drawn from geometry to be found in the poetry of long ago.

Just to read and enjoy – with no analysis - a poem of John Donne's (1572-1631) and one of Andrew Marvell's (1621-1678). Both poets use images taken from geometry to enhance the beauty/delight of their celebrations of love.

The Definition of Love

My Love is of a birth as rare
As 'tis for object strange and high:
It was begotten by despair
Upon Impossibility.

Magnanimous Despair alone
Could show me so divine a thing,
Where feeble Hope could ne'r have flown
But vainly flapt its Tinsel Wing.

And yet I quickly might arrive
Where my extended soul is fixt,
But fate does Iron wedges drive,
And alwaies crouds it self betwixt

For fate with jealous Eye does see
Two perfect Loves; nor lets them close;
Their union would her ruin be,
And her Tyrranick pow'r depose.

And therefore her Decrees of Steel
Us as the distant Poles have plac'd
(Though Loves whole World on us doth wheel)
Not by themselves to be embrac'd.

Unless the giddy Heaven fall
And Earth some new Convulsion tear;
And, us to joyn, the World should all
Be cramp'd into a *Planisphere*.

As Lines so Loves *oblique* may well
Themselves in every Angle greet:
But ours so truly *paralel,*
Though infinite can never meet.

Therefore the Love which us do bind.
But Fate so enviously debarrs,
Is the conjunction of the Mind,
And Opposition of the Stars.

Andrew Marvell

The planisphere is the projection of half the globe on to a plane – an astrolabe, a round plate the two sides of which show the two hemispheres, thus bringing the two poles together

A Valediction: forbidding mourning

As virtuous men pass mildly'away,
Andwshisper so their soules, to goe,
Whilst some of their sad friends doe say,
The breath goes now, and some say, no:

So let us melt, and make no noise,
No tear-floods, nor sigh-tempests move,
T'were prophanation of our joyes
To tell the layetie our love.

Moving of th'earth brings harmes and feares,
Men reckon what it did and meant,
But trepidatin of the spheares,
Thgh greater farre, is innocent.

Dull sublunary lovers love
(Whose soule is sense) cannot admit
Absence, because it doth remove
Those things which elemented it.

But we by a'love, so much refin'd,
That our selves know not what it is,
Inter-assured of the mind,
Care lesse, eyes, lips and hands to misse.

Our soules therefore, which are one,
Though I must goe, endure not yet
A breach, but an expansion,
Like gold to ayery thinnesse beate.

If they be two, they are two so
As stiffe twin compasses are two,
Thy soule the fixt foot, makes no show
To move, but doth, if th'other doe.

And though it in th'center sit,
Yet when the other far doth rome,
It leanes, and hearkens afetr it.
And growes erect, as it comes home.

Such wilt thou be to mee, who must
Like th'other foot, obliquely runne;
Thy firmness makes my circle just,
And make me end where I begunne.

John Donne

New materials from the Primary National Strategy:

Supporting children with gaps in their mathematical understanding

Following an extensive pilot and completion of a review of research on what works for children with mathematical difficulties, the Primary National Strategy has recently published some resources to support provision for children with difficulties in mathematics. **Mary Clark** describes these resources and their development.

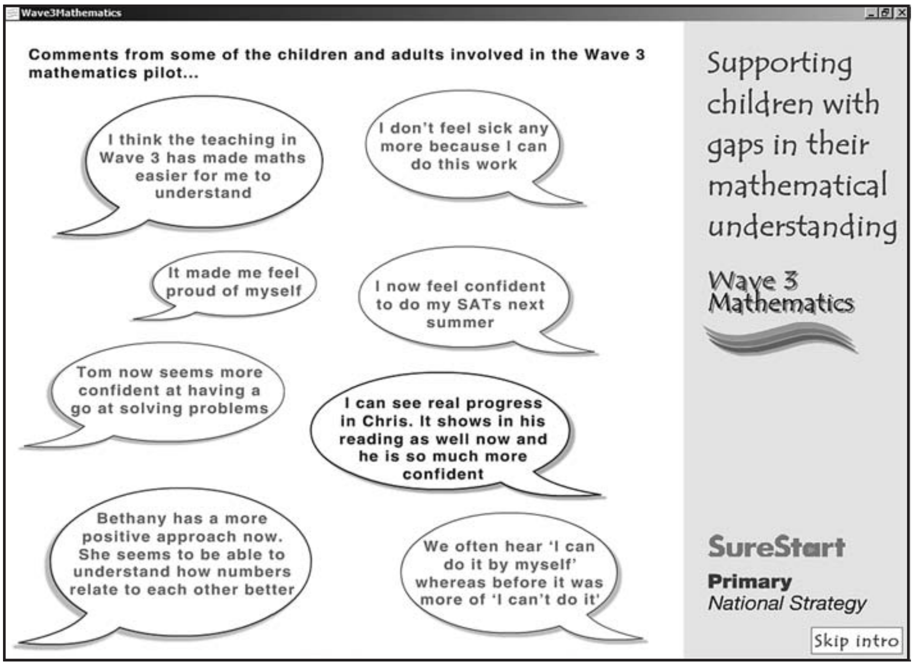
This article gives some information about these new Wave 3 mathematics resources, *Supporting children with gaps in their mathematical understanding,* and the background to their development.

Wave 1 The effective inclusion of all children in high quality learning and teaching of mathematics in the daily mathematics lesson.

Wave 2 Additional time-limited provision in the form of small-group intervention to accelerate progress and enable children to work at age-related expectations.

Wave 3 Additional time-limited provision to enhance the progress of identified children where Waves 1 and 2 are not, on their own, having the desired effect. This will involve focused teaching activities which tackle fundamental errors and misconceptions that are preventing progress.

The image below, which is taken from the CD-ROM from the pack, displays quotes from some of the children and adults involved in piloting draft materials during the pilot.



Children often expressed a growing confidence in their ability to do mathematics. This was achieved through time from a teacher or teaching assistant committed to working with them on an identified difficulty. Some children went as far as including reference to physical feelings that learning or not learning mathematics had invoked: “It made me feel happy. It gave me a warm feeling in my tummy because I could do it. It helped me to do division because I couldn’t do it and now I can.” This was a comment from Year 4 boy. Parents and carers noticed changes: “Jo will do her homework on her own now, before I get a chance to help her.”

Twenty-seven LEAs volunteered to take part in the pilot and in each of these about ten schools piloted the materials. During the two years of the pilot many others obtained the materials through the pilot website. Feedback came via LEAs and direct from pilot schools as well as via the website. A conference held for pilot LEAs in January 2004 was particularly helpful in reviewing progress and collecting many formative comments. Then a new phase in the writing began with ideas from the conference influencing the evolution of the design. Improvements were made to the assessment for learning opportunities throughout the materials, links to whole class work were added and activities in the form of games both for use during teaching sessions and to involve parents and carers were incorporated.

Schools tried a range of ways of working to include mathematics Wave 3 provision as they trialled the pilot materials.

There was enthusiasm generated through involvement in the pilot with schools reflecting on their practice and expressing commitment to continue developing this. These are some comments from schools in the pilot:

I think we might need to rethink our assessment records so that there is more space for the richness of the observations we can do in Wave 3.

We're going to do more TA training.

My advice to anyone setting out to do Wave 3 would be to keep asking yourself 'Am I doing this the best way?' We kept changing what we did in the light of circumstances and we're now on our third way of doing it … The results have been fantastic – more confidence and raised self-esteem and kids smiling and wanting to do maths.

Keep doing it! It's great!

We want to involve the parents more.

Anything that gets the kids this excited about maths has to be worth putting lots of time and effort into. We just want to do more of it next year.

Review of research on what works for children with mathematical difficulties

The DfES commissioned a review of research from Ann Dowker , University of Oxford[1], to identify what works for children with mathematical difficulties. This was published in 2004.

The research review suggests that:

- mathematical difficulties are common, often quite specific and show considerable individual variations;
- they are equally common in boys and girls, in contrast to language and literacy difficulties which are more common in boys;
- children's mathematical difficulties can take several forms. The causes for such difficulties are varied and include, for example, individual characteristics, inadequate or inappropriate teaching, absence from school resulting in gaps in mathematics learning, lack of preschool home experience with mathematical activities and language;
- children with mathematical difficulties typically combine significant strengths with specific weaknesses;
- some children have particular difficulties with the language of mathematics;
- difficulty in remembering number facts is a very common component of arithmetical difficulties, often associated with dyslexia;
- some children can remember many number facts, but seem to lack strategies (including suitable counting strategies) for working out calculations when they do not know the answer, other children show a pattern that is the reverse of this;
- other common areas of difficulty include word problem solving, representation of place value and the ability to solve multi-step arithmetic problems.

The research review concludes that:

- children's difficulties with calculation are highly susceptible to intervention. These interventions can take place successfully at any time and can make an impact;
- individualised work with children who are falling behind in number and calculation can have a significant impact on their performance;
- the amount of time given to such individualised work does not, in many cases, need to be very large to be effective. Short but regular interventions of individualised work may bring a child to the point where they can profit much better from the whole-class teaching that they receive;
- it is important to find out what specific strengths and weaknesses an individual child has; and to investigate particular misconceptions and incorrect strategies;
- interventions should ideally be targeted towards an individual child's particular difficulties. If they are so targeted, then most children will not need very intensive interventions.

Design of the published Wave 3 materials, *Supporting children with gaps in their mathematical understanding*

Messages from the review are reflected in the design and features of the Primary Strategy materials. The focus of the materials is on number and calculation with highlighted features of problem-solving (Using and applying mathematics) in each teaching unit. The materials set these specific areas of mathematics within everyday contexts and wider mathematical contexts with the result that other parts of the mathematics curriculum such as measures are also included. The framework within which the teaching units are set is clearly one which supports teachers in identifying children's particular difficulties so that some well-targeted work can be done with the child. The intention is that the additional support is of relatively short duration.

The teaching materials evolved as feedback was provided by those piloting the draft materials and in response to the research. The guiding principles informing the design are:

- flexibility so that teachers can adapt them;

- sharing the purpose of each activity with the child to encourage reflection on, and ownership of, learning;
- highlighting and modelling key vocabulary throughout;
- teaching activities finishing with related activities for whole class use, where appropriate;
- use of a variety of images and models aiming to include some the child may not have met before;
- linking mathematics to familiar and relevant contexts;
- integrating and exemplifying mathematical problem solving;
- inclusion of games amongst teaching activities possibly for sharing with parents and carers.

Particular mathematical themes are fundamental to the design of the teaching units:

- Using and applying mathematics has been integrated. Often there are several opportunities for problem-solving within one activity, but in each one particular opportunity has been highlighted.
    - Encouraging children to discuss and explain in order to support development of their mathematical reasoning;
    - Opportunities for children to make choices are woven into the activities, for example, selecting numbers, devising calculations
    - Encouraging children's own recording to communicate mathematical thinking, focusing on efficiency;
    - Opportunities for evaluating the efficiency of methods of calculation.
- The importance of mathematical language for the child's conceptual development is emphasised and key vocabulary is listed in each activity. It is important for adults to use correct mathematical language and to facilitate this, examples are given in words, for example, 725 x 3 is accompanied by what the adult could say to the child: 'Seven hundred and twenty five **multiplied by** three'.
- There is a focus on progression in counting from the earliest stages through to Year 6 to support the development of secure counting skills.
- Throughout the materials there is emphasis on the process of estimating first, then calculating and finally checking.
- Decimals are addressed within meaningful contexts, for example, via displays on a calculator and as a part of measure.
- Structured equipment and everyday materials are used to model mathematical concepts, supporting children's mathematical thinking and development of mental imagery. Some links to ICT resources such as the Primary National Strategy Interactive Teaching Programs (ITPs) are included.
- A wide range of resources is used in the teaching sessions. Teachers' selection of these to suit the needs of their children is an important part of adapting the materials.

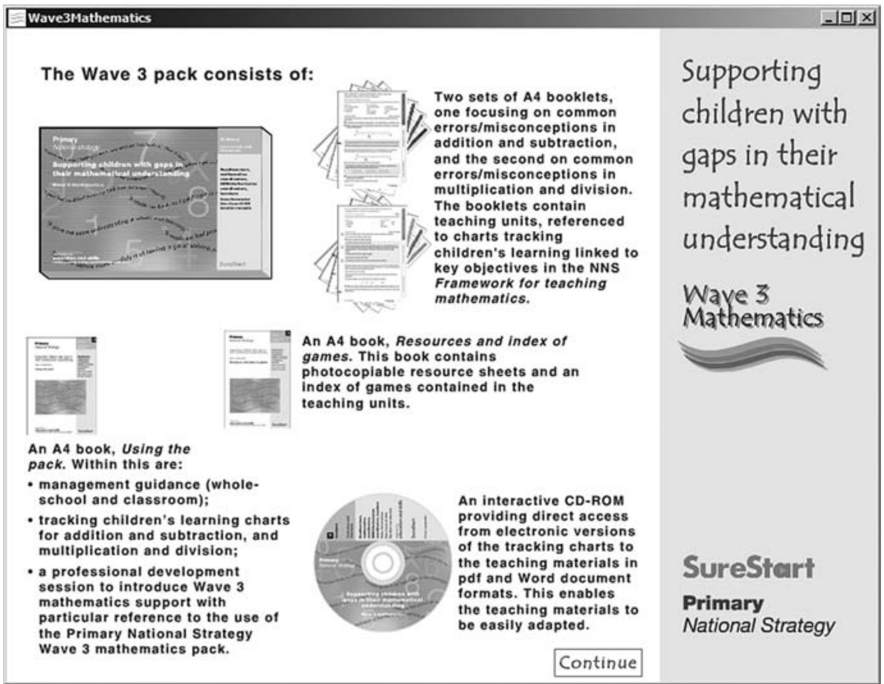
The materials reflect best practice in assessment for learning as a key tool for raising achievement, through

- use of questions to elicit information about children's understanding;.
- sharing the purpose of the activity with the learners;
- encouraging children' s reflection on their learning and identifying for themselves possible next steps.

Revision after the pilot and publication

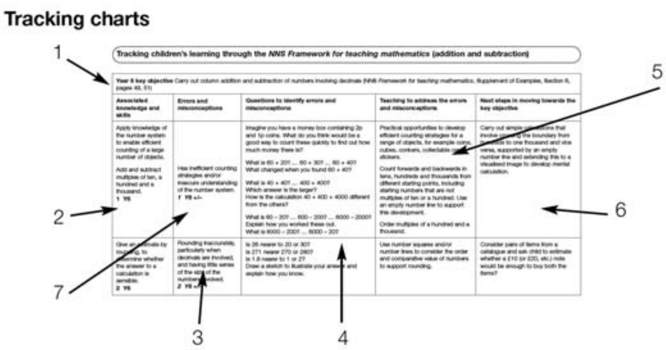
Following the pilot phase the materials and the tracking children's learning charts, which provide the framework to which the teaching units are referenced, were revised. The tracking charts, one for addition and subtraction and one for multiplication and division, have been revised to improve their clarity and accessibility for busy teachers. The contents of the teaching units have been enhanced to ensure that they all reflect the design features listed above, in particular adding to the models and images used to support children's learning.

The diagram below, taking from the interactive CD-ROM included in the pack, shows the components that make up the complete *Supporting children with gaps in their mathematical understanding* pack:



The diagram below, taking from the interactive CD-ROM included in the pack, shows the components that make up the complete *Supporting children with gaps in their mathematical understanding* pack:

Following the success of the interactive tracking children's learning chart and the downloadable teaching materials which were available on the pilot website, the CD-ROM reflects these particular popular elements of the pilot website. The interactive tracking chart supports linkage between the error/misconception listed on the chart and the teaching unit matched to this teaching focus. Errors and misconceptions are matched to a progression in addition and subtraction, and one in multiplication and division, from Year R to Year 6. The year groups are used merely as labels to give an indication of the approximate position in the progression through the mathematical stages represented in the NNS Framework for teaching mathematics. The chart forms the teacher's starting point in deciding on the area of difficulty that needs to be tackled based on the teacher's day-to-day assessment of the child's strengths and weaknesses. The diagram below shows an example page from a tracking children's learning chart.



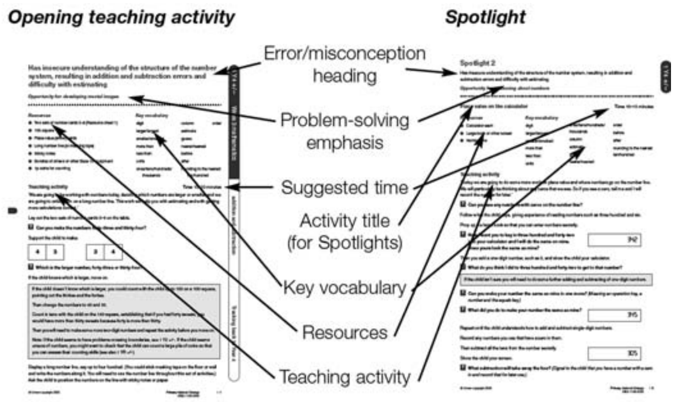
1. Year group key objective.
2. This column lists associated knowledge and skills that contribute to understanding of the year group key objective.
3. Common errors and misconceptions linked to specific knowledge and skills are listed to support diagnosis of children's difficulties.
4. Questions in this column can be used to help the teacher decide on where the child's difficulties lie.
5. Examples of the types of teaching activity in the A4 booklets.
6. This column provides ideas to develop when the child has improved their understanding of the identified difficulty. The teacher can make use of these ideas to consolidate understanding and extend thinking.

Teaching units are provided in hard copy in the pack. To make it simpler for teachers to amend teaching sessions to better tailor them for their children Word versions of the teaching units can be taken from the CD-ROM and adapted. All the resource sheets are stored on the CD-ROM as well as being in printed format in a book in the pack. Resource sheets contain cards, game boards etc rather than worksheets. Children in the pilot noted with pleasure the lack of worksheets and relished the opportunities they had to talk through their work in a one to one or small group with their teacher or teaching assistant.

The structure of each A4 teaching unit booklet:

- Focus error/misconception
- Opening teaching activity addressing error/misconception
- A number of Spotlights (short focused teaching activities from which to select). It is intended that these are used in short teaching sessions to help reinforce understanding.
- Final Spotlight which includes assessment opportunities, often encompassed in a game, key vocabulary check list, and intended learning outcomes list.

Linked resource sheets are referenced where appropriate.



In order to support the range of staff who may be deployed to work with an individual child or small group using these teaching units the activities are written to include some support for the user as shown in the extract at the end of this article.

Supporting children with gaps in their mathematical understanding was published in April 2005 and many LEAs are currently in the process of working with their schools to launch the pack. If you are interested in finding out more the pack is available for schools to order free of charge from Prolog (Tel:0845 60 22260), quoting reference DFES 1168-2005 G.

The materials can also be viewed on the Primary Strategy website at www.standards.gov.uk/primary.

1. What works for children with mathematical difficulties (DfES Research report 554), available from DfES Publications (tel: 0845 60 222 60) or can be downloaded from the website www.dfes.gov.uk/research

Equals at BCME 6

Four members of the *Equals* editorial team led a session at the recent British Congress of Mathematics Education conference. The work of some of the participants is reported here.

As an introduction we spoke about the difficulties of giving pupils with special educational needs an appropriate diet of mathematics. We said that all pupils should be given the opportunity to explore mathematics (as opposed to just concentrating on 'the basics') and that this meant:

- High expectations
- Creating the right classroom climate, including concrete, relevant, visual materials and an atmosphere where it is okay to make a mistake
- Opening doors and windows
- Giving pupils space and time
- Designing activities which have more than one answer
- Diagnosis and then moving the mathematical understanding on

In talking about continuing professional development (CPD) we said:

- It can't be a one off; participants need some input, time to try things out in school and then return for reflection
- It is a dynamic process where participants need to be active, not just listening (they need to **do** the mathematics)
- It needs a supportive structure back at school where they have colleagues who can share in the successes and the obstacles

Participants were then asked to plan a mathematical activity/teaching sequence which:

- could be used in a CPD session with teachers (or teaching assistants)
- could be taken back to the classroom tomorrow
- has an open start, and can be developed at different levels
- demonstrates how to plan for differentiation generally i.e. is not just a designer one-off
- uses multilink or unifix cubes (optional)

To support them various suggestions of starting points were given. They worked in groups of between 2 and 5 and some of their activities appear below.

Two activities were variations on 'finding all the possibilities'. Other activities will appear in future editions of *Equals*. Where we feel it is helpful we will give the starting point they used as this illustrates the adaptation which they undertook to provide the differentiation and progression for the activity. We invite you to try out the activities at a level appropriate to your class and report back on the outcomes.

Objectives for these first activities can be found in the Primary Framework from years 2-6 ('Solve mathematical problems or puzzles, recognise and explain patterns and relationships, generalise and predict. Suggest extensions by asking "What if…?" ') and year 7 ('identify all the possible mutually exclusive outcomes of a single event.')

Ice cream shop

The ice cream shop sells 3 different flavours: Orange (O), Vanilla (V) and Lime (L). (These flavours were chosen to correspond with the colours of the multilink cubes they had.)

If I am allowed to choose a single scoop ice cream:
How many choices do I have?

(This can be modelled using cubes, or some other recording method [O, V, L])

If I am allowed a double scoop ice cream.
How many choices do I have now?
Can you make/represent them?

OO	OV	OL
VV	VL	
LL		

What about a triple scoop?
Is there a pattern?
Does it continue for 4 and 5 scoops?

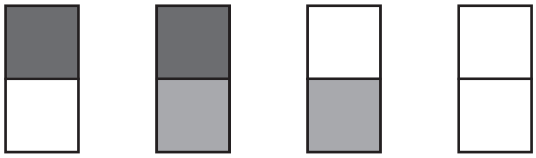
Extension: The shop now sells 4 flavours…..

Towers

Resources

For this activity you need unifix rather than multilink as the towers need to have a unique way up.

You have 2 colours of unifix cubes, how many different towers can you make which are 2 cubes high?
Model this with the class and draw the possible combinations on the board:



What if you can build towers 3 cubes high; how many different ways are there now? [8 possibilities]

This can be extended either by staying with 2 colours and making the towers higher, or by adding colours. Ask the class 'How could we make this more difficult?' and take their suggestions.

Note the patterns and ask them to predict what will happen if the tower can be one cube higher or if we have another colour.

Reviews A Book Tasting by Rachel Gibbons

There are two books I would like to draw to your attention and I have decided that, rather than writing about them in the usual review format, I would invite you to a "Book Tasting" which might better give you their flavour and encourage you to savour them further. They are vastly different pieces of writing with quite different purposes. One is a book full of research findings by well-known researchers of the mathematics classroom. The other is full of tips about how to do simple arithmetic, which does at first look like the old textbook full of various mysterious instructions followed by practice exercises and precious little else. But on a second look the 'else' in this case is of great value though it would probably be preferable to describe the "tricks" as calculation strategies rather than tricks which seems to imply low expectations of being able to understand them.

This book is very old by today's standards, but the story of how I came across it seems worth sharing. When I had made some purchases in a small stationers the young manager totted up the total on a calculator. Nothing strange so far, but he did not stop there. He was scribbling on a scrap of paper. 'You're not checking the calculator are you? I asked. 'Most people couldn't do that. You must be interested in numbers.' 'Oh, yes I am' he said. He told me he had a very interesting book about calculating tips that he studied, indeed he had it in the shop and very happily fetched it so that I could judge it for myself. It was

Rapid Math Tricks and Tips: 30 Days to Number Power **by Edward H. Julius**, New York: John Wiley & Sons, 1992

Edward Julius starts by welcoming readers to the fascinating world of rapid calculation. Here are some of the tips and tricks he presents.
From the introduction:

The more energy you focus on "number power" the greater the likelihood that you will master the techniques presented. Your ultimate reward will be their practical application. …

To become a number-power wizard you don't have to be a descendant of Albert Einstein, work as a rocket scientist or possess an advanced degree in differential calculus. All you need is a basic understanding of addition, subtraction, multiplication and division. …

The Test of Reasonableness (T of R)

In general, applying a test of reasonableness to an answer means looking at it in relation to the numbers operated upon to determine if it's "in the ballpark". Put in simple terms, you look at the answer to see if it makes sense. For example, if you determine 10 percent of $75 to be $750 (which could happen if you use a calculator and forget to press the percent key), you should immediately notice that something is very wrong.

The majority of the number-power tricks in this book involve multiplication and division. As you will learn very shortly, you should ignore the decimal points and the zeros when starting the calculations. When you've completed the calculation, you will need to determine if and where to insert or affix a decimal point or zeroes based upon "what looks right". …

Week 1 **Multiplication and Division**

Day 1

Before you begin the program with today's techniques, make sure you understand the basic math concepts reviewed previously. In particular, you should be able to multiply and divide by 10, 100, and so forth, as illustrated in items 5 and 6 on page 8. Today's tricks are very important because they will be used as building blocks for many of the later tricks.

4,800 ÷ 120

Step 1 Disregard the zeroes and think "48 ÷ 12"

Step 2 Divide 48 ÷ 12

Step 3 Apply T of R: as explained previously, you may cancel an equal number of right-hand zeroes when dividing. Therefore, the problem becomes 480 ÷ 12. we know that 48 ÷ 12 = 4, so 480 ÷ 12 must equal 40 (the answer).

Thought process summary

4,800 ÷ 12 ➛ 48 ÷ 12 = 4 ➛ 40

This leads to the following 22 days later:

Day 23

Trick 45: Rapidly Divide by 8 (or 0.8, 80, 800, etc.)

Strategy: …To divide a number by 8, multiply the number by $1\frac{1}{4}$, and affix or insert any necessary zeroes or decimal points. The easiest way to multiply by $1\frac{1}{4}$ is to take $\frac{1}{4}$ of the number and add the number itself…

…

Brain Builder # 2

1.8 ÷ 0.8

Step 1: Disregard the decimal point and think " 18 ÷ 8"
Step 2. Multiply: 18 x $1\frac{1}{4}$ = 22.5 (intermediary product).
Step 3. Apply T of R: A quick estimate puts the answer just above 2.
Step 4. Move the decimal point of the intermediary product one place to the left, producing the answer 1.25

Thought Process Summary

1.8 ÷ 0.8 ➛ 18 ÷ 8 ➛ 18 x $1\frac{1}{4}$ = 22.5 ➛ 2.25

And now the book of research findings:

Ian Thompson ed., *Enhancing primary mathematics teaching*, Maidenhead: Open University Press, 2003

* This chapter will examine the recommendations of the National Numeracy Strategy regarding assessment. These suggestions will then be developed in the light of further research….
Black and Wiliam discussed … how formative assessment could be improved. The findings are reported under four headings:

Questioning. Teachers need to ask questions requiring more thought and to allow appropriate time for children to consider and formulate their response. This can sometimes be done with a partner or a small group. All children are expected to contribute an answer even if it is incorrect. This allows teachers to get a clear picture of the children's understanding in order to give immediate feedback. …

(pp. 101, 107 Rosemary Hafeez, 'Using assessment to improve teaching and learning.')

……………………………………

* … we believe that there are two main areas where calculators are at their most effective in the learning of mathematics, and these apply to all key stages:

as a calculating aid in solving problems: to 'free' children from mundane calculations, allowing them to concentrate on the process they need to employ to solve a problem; to help them 'keep their eye on the ball'

as a teaching aid: a rich piece of equipment that can be used to illuminate and model how our number system works.

(pp. 155,6 Helen J Williams and Ian Thompson, 'Calculators for all?')

……………………………………

* There is relatively little hard evidence for any beneficial effects of ICT on mathematics in the primary age range. …

Information can be manipulated easily on a computer so that the pupil can make changes and evaluate the effect of those changes. Observing in a graph when changes are made to the table of numerical information on which the graph is based, or by manipulating an algebraic formula and observing how a graph of that function changes on a computer or graphical calculator can develop pupils' understanding of mathematical relationships.

(p. 169 Steve Higgins, 'Does ICT make mathematics teaching more effective?')

……………………………………

* … the understanding of place value in both whole numbers and decimals is limited for many pupils even in Key Stage 3' (Askew and Brown) …
Consider the following examples of mental strategies used by young children…and think about the aspects of value knowledge that are utilized in each case:

- *John, aged 9 (35 + 27): Well, 30 and 20 is 50 … 5 and 7 is 12 … and add 50 and 12 makes 62.*
- *Sophie, aged 9 (54 – 27): 54 take 20 is 34 … and 34 take 4 gives me 30 … if I take the 3 from the 30 I've got 7, I mean 27.*
- *Elizabeth aged 8 (28 x 5): 140 … I put 20 times 5 would be 100 and 8 times 5 is 40 ... because I know tables and that's how I found out.*
- *Emma, aged 8 (46 ÷ 2): 23 … half of 40 is 20 and half of 6 is 3 … plus 20 and 3 is 23.*

(pp. 181,184 Ian Thompson, 'Place value: the English disease?)

……………………………………

* Both observation of lessons, … and interviews with children suggest that low attaining pupils derive little benefit from whole-class teaching episodes, and the topic of the lesson does not always correspond to their areas of greatest need. Some high attainers also expressed to us their frustration at their progress being held back by the whole-class teaching emphasis, which tends to be pitched at the needs of the middle of the group.

(p. 202 Margaret Brown and Alison Millett, 'Has the Numeracy Strategy raised standards?')