Equals

mathematics *and* special educational needs

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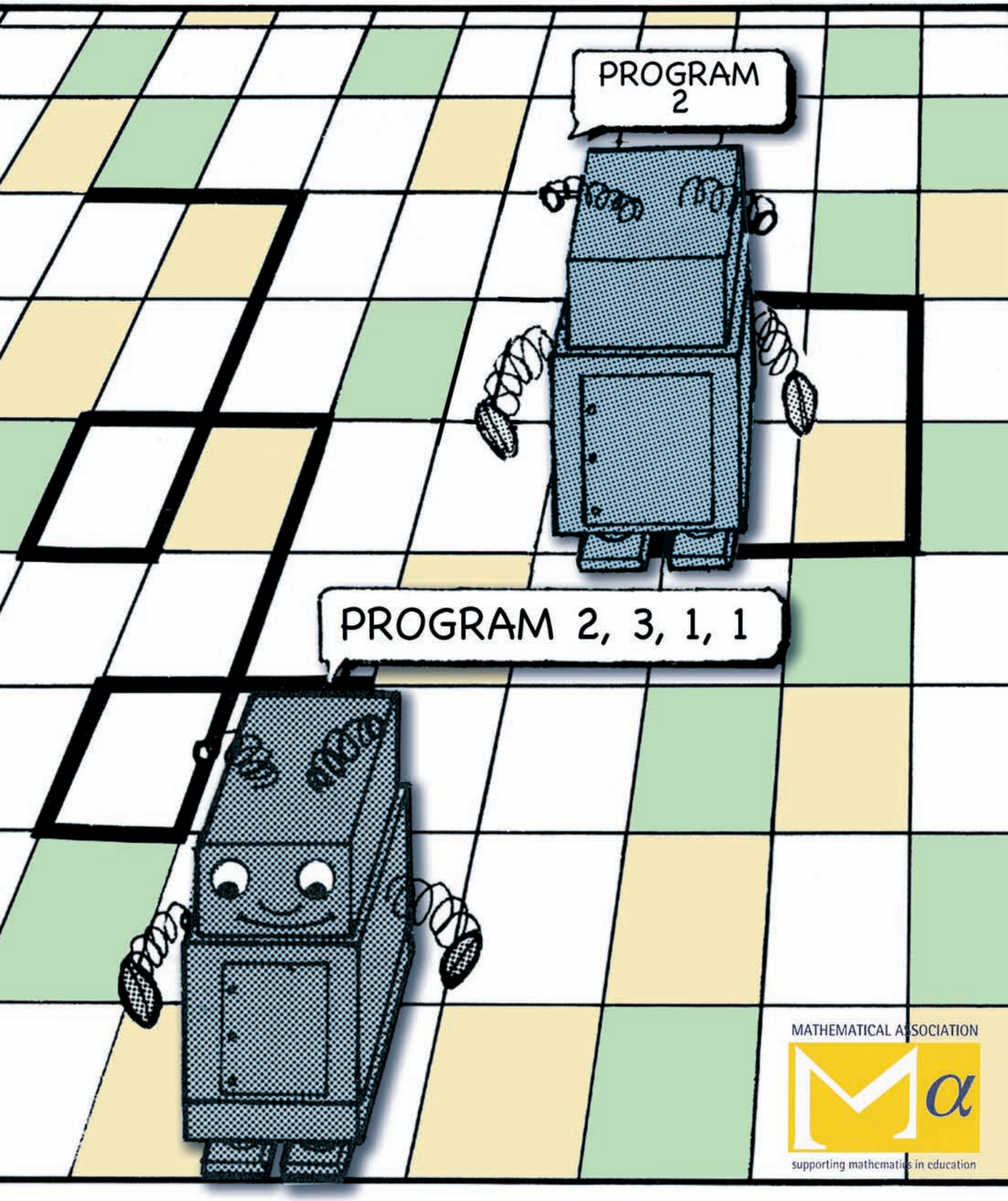

MATHEMATICAL ASSOCIATION
supporting mathematics in education

Equals mathematics and special educational needs

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Editorial

Creating drop-outs

So David Burghes thinks that making maths optional at 14 would solve many problems in the world of mathematics education.[1] Such a suggestion from the director of a *centre for innovation* in mathematics education seems almost inconceivable. It is hardly an innovation rather a harking back to some of the bad old practices of the past when the "low attainers", who were not considered worthy of mathematics proper, did "civic arithmetic" for as short a time as possible. Burghes apparently considers that his suggestion, if followed, will lead to better motivated classes. Of course, if you remove those who have little or no interest in mathematics you will have less trouble in motivating those who are left. But it is as well to question whether those who would, if allowed, drop the subject have been turned off because they are incapable of appreciating it or because no one has ever shown them its excitement.

Motivating mathematics

There is excitement for all to be found in mathematics. As Margaret Brown says in her opposition to Burghes's proposal, "it is quite possible to teach maths in an interesting way". This possibility is demonstrated in the following pages, particularly in "Living Graphs" where we read of the innovative ideas of an NQT which clearly motivated the class.

The linking up of "Music Mathematics and Language" can make for excitement, as can taking a historical perspective to begin to answer the question "Why Measure?" while "Loop the Loop" brings out so many of the "what if" questions that are essential to any meaningful mathematical journey. Should we deny some pupils the possibility of such mathematical journeys? We agree with Brown that in those "who would elect to stop maths early – the bias is likely to be towards girls and less-advantaged students." These are the very students who will be even more disadvantaged, not only because of a lack of job opportunities but because of a stunted understanding and appreciation of the world in which they live because they lack the language in which so much science and technology is expressed. If some teachers too are short on that language they must be supported to become more fluent. In this the writing of both Gerry Rosen and Stewart Fowlie can help by showing a little of the mathematics underlying some quite simple classroom activities. A greater understanding of the difficulties some students will experience, some of which are explored by Arno Rabinowitz, is important. "Ratio for All" gives further insight into mathematical concepts and how they ar acquired by children.

Sharing ideas

But, as Brown confirms, the excitement cannot be engendered in a world "dominated by tests and league tables, over-full and outdated content, poor quality teaching materials rushed out untrialled for changing syllabuses and under-educated teachers". Working and learning together is vital if we are to make mathematics lessons more exciting for all. "The Excitement of Working Together" gives an instance of teacher education that is home-grown and inexpensive. *Deep Progress in Mathematics*, reviewed in this issue, maintains that if they are to be brave enough and clever enough to work in a way that benefits their students, even if it is at odds with decreed practice, teachers will find support in working with others over time, "as you can do in a research project or professional development".

Extra information, corrections, etc.

A reader asked us how to contact Annette Glatter. Those equally enthused by her approach to teaching and more details of her mathematical games can contact Annette by e-mail at mathletts@hotmail.com

Apologies to Gerry Rosen for the typing error in diagram 6 of his article in 10.2. The last statement should read (b + b) – (b + g).

[1] *TES*, 13 August 2004

Research this week suggested parents will spend £1.3 billion on essential items for their children ahead of the new school year. The average parent will fork out around £132.84 on uniform, PE kit and footwear for their offspring, according to Capital One.
Parents with boys can expect to splash out more than those with girls, with boys' uniforms and shoes costing an average of £138.02 compared with £127.66 for girls.
News in brief, *TES*, 3 September 2004

More than a million children in Britain are looking after someone with an illness, disability or health problem who could not manage alone.
News in brief, *TES*, 3 September 2004

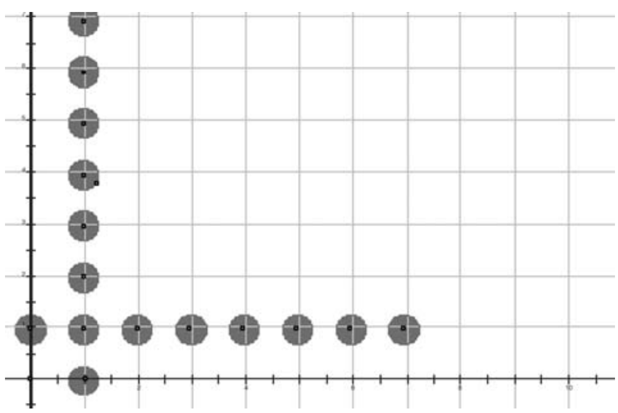
Living Graphs

The newly qualified can have much to offer in the way of innovative ideas as **Gill Emerson** discovers

I was invited to observe a Y8 lesson on straight line graphs planned and taught by Jonathan Robinson, a NQT, in January 2004.
The venue was the school hall. Prior to the start of the lesson 64 chairs were arranged and labelled to represent the co-ordinate grid $0 < x < 7$, $0 < y < 7$. A TV and video camera were set up on the stage, so the pupils could see what was happening on the floor of the hall without turning round. Two maths groups were brought into the hall and told to sit where they liked. Some pupils chose to sit with friends from the other maths group. The other maths teacher and three teaching assistants were also present. The end of KS2 levels for the two groups were L4/5 and L3/4.

Jonathan began the lesson with a recap of the vocabulary that would be used during the lesson: x-axis, y-axis, co-ordinates, axes, gradient, origin, point of intersection, intercept; pupils were invited to explain what they understood by each of the terms.

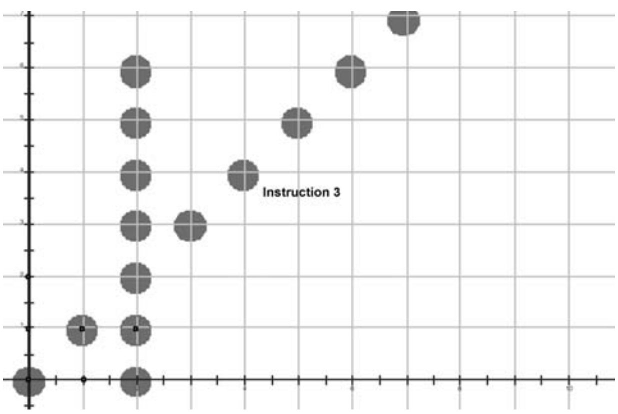
The main part of the lesson began with Jonathan instructing the pupils forming the x- and y-axes to stand up: "Stand if your x-coordinate is zero or your y-coordinate is zero." The pupil representing the origin was identified and the pupils were encouraged to look at the TV screen and all around them. During the next 35 minutes or so the pupils were given 13 sets of functions to graph and particular features to identify and explain.



1. (See illustration above) "Stand if your x-coordinate is one or your y-coordinate is one." Once the pupils had agreed who should be standing …
Q: *What are the equations of the two lines?* Pupils were given time to think and discuss before answers were taken.

"Everyone except the point of intersection of the two lines sit down." One pupil was left standing.
Q: W*hat is the point of intersection, and why?* Again, the pupils were given time to think and discuss before answers were taken. The point of intersection was invited to sit down.
Q: *How could you describe the line x=1?* [Seeking 'vertical'] *How could you describe the line y=1?* [Seeking 'horizontal']

2. "Stand if your y-coordinate is four or your x-coordinate is three."
The pupils formed the lines more quickly and the questioning was the same as in 1.



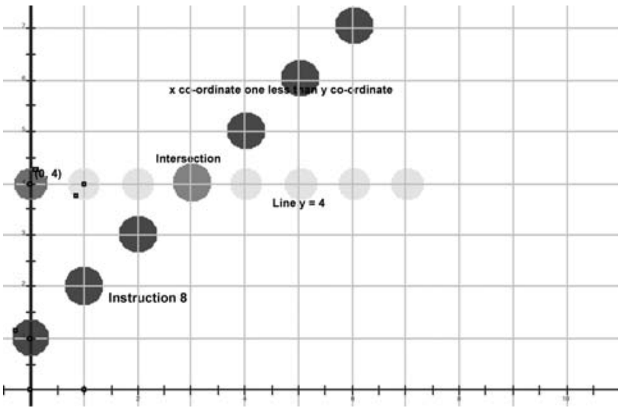
3. (See illustration above) "Stand if your x- and y-coordinates are the same, or your x-coordinate is two."
There was more discussion between the pupils about who should be standing. Having correctly identified the equations of the two lines and the point of intersection, the pupils were asked to explain what is the same and what is different about the two lines. [Seeking 'steepness' and/or 'gradient' in the explanations.]
4. "Stand if your x-coordinate is one bigger than your y-coordinate or your x-coordinate is five."
After some discussion the pupils correctly formed the two lines and the questioning was the same as in 3.

5. "Stand if your y-coordinate is two more than your x-coordinate or your x- and y-coordinates are the same."
This time the questioning was designed to draw out the intercepts on the y-axis and that the lines are parallel.

The next three sets of instructions focused on the pupils forming straight lines that pass through a given point of intersection.

6. “(4,3) stand up – you are the point of intersection. [Pause while pupil stood up.] Stand if you are on a vertical line through the point of intersection.”
Q: *What is the equation of this line?*
“Stand if you are on a horizontal line through the point of intersection.”
Q: *What is the equation of this line?*

7. “(1,1) stand up – you are the point of intersection.” Instructions and questioning as for 6.



8. (See illustration above) “(0,4) stand up – you are the marker; look along the line y=4. [Pause while pupil stood up.] Stand up if your x-coordinate is one less than your y-coordinate and you intersect with the line y=4.”
The second instruction provoked a lot of discussion amongst the pupils. Once they agreed who should be standing the marker was asked to state the coordinates of the point of intersection. This caused some consternation and a lot of whispering because the marker had forgotten the second instruction! Help was at hand and the correct answer given.

The pupils’ reactions to the next two sets of instructions began with bewilderment, followed by animated discussions, and ended with some, but not all, being convinced.

9. “Stand up if your y-coordinate is double your x-coordinate.”
Eventually, pupils with coordinates (1,2), (2,4) and (3,6) stood up.
Q: *What is the equation of this line?* A tentative “Is it y=2x?” was offered after much discussion.
Q: *What are the coordinates of another point on this line?* This prompted answers of (4,8), (5,10), etc, but not the expected (0,0).
“Stand up if your coordinates are on the line y=x.”
Q: *What is the point of intersection of these two lines?* (0,0) was offered, and agreed by the pupils.
Q: *So, what are the coordinates of another point on the line y=2x?* (0,0) was offered and agreed by the pupils.
Q: *Which is steeper, y=x or y=2x?*
This proved to be a tricky question for two reasons:
a) although the pupils had shown some understanding of gradient/steepness earlier in the lesson, many were unclear about the meaning of ‘steeper’ in this context
b) the camera angle, while not quite parallel to the hall floor, was insufficient to discriminate between the two lines of pupils.

In the end the pupils decided that y=2x is steeper.

10. “Stand up if your y-coordinate is one or your y-coordinate is four.”
Having established that two parallel horizontal lines had been formed, the next instruction, “People with the highest intercept value remain standing, the rest sit down,” had them bobbing up and down while the rest of the pupils offered advice. Listening to the pupils’ advice it was clear that some were confused between the terms ‘intercept’ and ‘point of intersection’ and one bright spark pointed out that you can’t have the ‘highest’ of two! Eventually they agreed that y=4 was the required line.

The last three sets of instructions involved straight lines with negative gradients. Up to this point none of the pupils had met the idea of a negative gradient.

11. “Stand up if your x- and y-coordinates add up to three or your x- and y-coordinates add up to five.”
Questioning drew out the equations of the lines as x+y=3 and x+y=5, the coordinates of the intercepts with the y-axis, the fact that the lines are parallel, and the observation that the lines ‘go the wrong way’.

12. “Stand up if your x- and y-coordinates are the same or your x- and y-coordinates add up to four.”
The questioning was intended to draw out the equations of the two lines, the point of intersection, the intercepts with the y-axis and how they can be recognised from the equations of the lines. All but the last part was answered correctly by the pupils.

13. The instructions involved pupils on the lines y+x=5, $y=\frac{1}{2}x$, y=0 and x=1 all standing up at the same time.
The pupils who were still seated saw a confused picture whether they looked around or at the TV screen. The final instruction: “People on the line with no gradient sit down” gave rise to confusion and arguments, and eventually a few pupils on y=0 sat down together with a couple on x=1 (because they had seen (1,0) sit down!).

The main activity was concluded with a reminder that lines like y=1 have 'no gradient', and a recap of 'point of intersection'. The pupils handed their coordinate cards to the TAs, stacked the chairs, and returned to their normal teaching rooms for a plenary session.

Although there was a chalkboard in the hall, nothing was written on it during the lesson. Apart from me, the only people who wrote anything during the lesson were the three TAs.

I accompanied the pupils from Jonathan's class back to their teaching room and led the plenary session. I had noted that a pupil in this group was 'the point of intersection (4,3)', so I wrote (4,3) on the board. The pupils were asked to suggest equations of straight lines that pass through (4,3) and then this was repeated for (3,5). I recorded their suggestions on the board so they could decide whether to accept or reject the equations. The end of lesson bell rang but the pupils were reluctant to leave because they wanted to continue.

This was a courageous lesson from a young NQT. The pupils helped each other; the more confident ones generally being the more able. Roughly equal numbers of pupils from each class volunteered answers and Jonathan took care to take answers from as many different pupils as possible. When questioned after the lesson, most of the pupils felt that they had a better understanding of straight line graphs as a result of the lesson.

Jonathan reflected on what he would do differently next time:

- Record the lesson on video tape!
- Position the video camera higher up and angled down, to give a better picture of the graphs.
- Involve the other maths teacher and the TAs in the organisation and delivery of the lesson.
- Use all four quadrants of the coordinate grid (this had been planned but the second set of coordinate cards had not been brought to the hall!).

Wokingham District Council
Education Service

Ratio for All!

Are difficulties with ratio due more to numeric presentations than to the concept itself? **Mundher Adhami** suggests that the visual handling of ratio-and-proportion problem may allow all to access the concepts intuitively before moving to addressing these number terms.

The reason that Number is difficult does not seem related to something inherent in the concepts themselves. Increasingly scientists are convinced that our brains are pre-wired to recognise numerosity and number relations. They see it as one of the universal human traits parallel and connected to language acquisition.[1] Any engagement with the physical world would seem to assure these mental capacities in people, with or without organised schooling. After all, societies throughout human history used and recorded numbers independently from each other, as evident in the ways the ancient Egyptian, Babylonian, Chinese, Romans, Arabs and many others have used numbers. Pre-historic/pre-settled communities, very small children, and even chimpanzees recognise quantities both in 'counting' terms, especially in small numbers, say up to 5, and in comparisons of size. But of course without some advanced social means of communication, and therefore social learning in general, this recognition does not go very far.

Children age 7 have shown they can intuitively deal with ratio and proportion, to the extent of describing such relations in numbers, almost formally in ratio form.

In our advanced cultures many properties of Number become accessible almost to all, with or without schooling. Some other properties remain problematic. It is worth looking at examples of these two kinds, and the reasons for ease or difficulty.
Of the more accessible kind are the **additive properties of number**, which require initially little more than being conscious of the learner's actions on collections of objects. Ability to count itself is a way of keeping track of a series of pointing actions using words so that a bowl of 7 apples comes to have a label of 7 for the whole collection, regardless of the order of counting, or differences of apple sizes.

Putting on top another collection of 4 apples clearly makes the resulting collection larger while taking away some apples makes it smaller. You can also split the 7 apples in many ways, and record all these actions on paper with comparatively little difficulty. From there on, the recording comes to have a life of its own, so doing calculations on a page can still be accessible to many, through being meaningful. Numerals are always interpretable as mere 1-to-1 labels of collections while place value conventions can be, with adequate teaching, interpreted as labels of units and collections-of-units in agreed sizes, e.g. 10s and 100s. Beads, the abacus, and Dienes blocks, make all these ideas tactile and beyond these the number line is still visible. In relation to additive properties, or additive composition of numbers the representations, or the 'cultural tools' for working on Number, do seem very useful. At least they do not cause too many problems.

Many problems, however, arise with numbers such as fractions and ratio, which sometimes are labelled 'multiplicative relations'. This term seems confusing since it leads one to think of the operation of multiplication as a procedure, memorised or not. I suggest this domain is better described simply as **relational**, since the object of attention in each case requires **handling a magnitude in relation to another**, or some quantity in relation to another. It seems sometimes that we create confusion by not attending to the relational essence of these numbers.

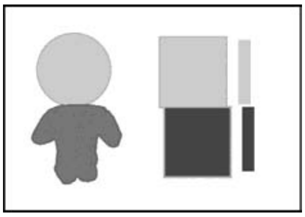
Recently in research in KS1,[2] we developed a lesson that proved successful in supporting the development of such relational thinking. Children age 7 have shown they can intuitively deal with ratio and proportion, to the extent of describing such relations in numbers, almost formally in ratio form. Teachers were surprised at how an appealing context, such as making sweets in two flavours, represented as pictures of 'jelly babies', have kept children engaged in demanding mathematics. It is a mathematics that is often expressed in natural language, but that is easily translated into numbers and even recorded.

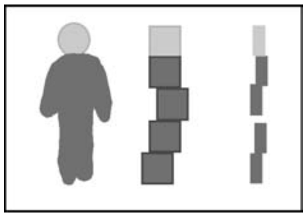
Considering the truly huge range of ability, i.e. processing capacity, use of language, and even dexterity of children at this age - and at any age - we claim that such an activity is useful for many older children. It can also serve as a first step for more advanced cycles of work in the same contexts, as we shall see at the end of this article.

Here is an account of the lesson activity.

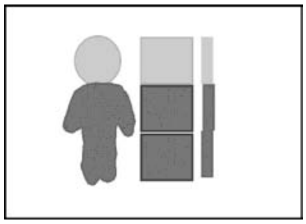
Episode 1: Comparing drawings

The teacher tells a story about a factory making jelly sweets in two flavours for head and body, and asks the children to choose the two flavours. She gives children two pictures, one showing a shape in two equal size parts for head and body. The other picture has the head as only a quarter of the body. Children find names for the two types in the context of the story (e.g. 'jelly baby' 'Jelly Daddy' or Jelly man) and find ways of talking about the sizes of the head and body in each to answer the question: *"What is different between them?"*





A third picture is then introduced. "How different is this?" would lead to a discussion on the use of blocks or rods placed on the sides of the pictures. A name should be found, e.g. 'Jelly Child' or 'Jelly girl', with the children arranging the pictures in order.



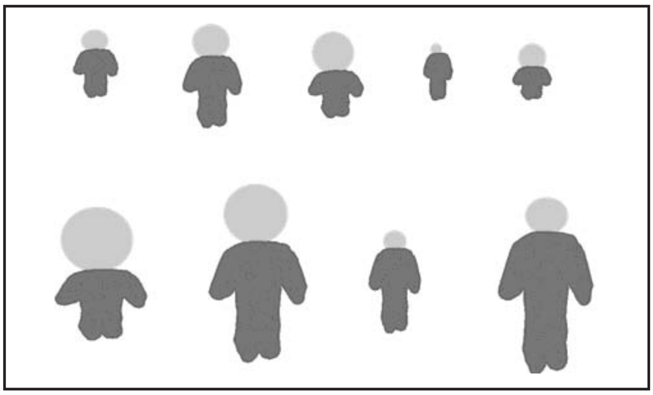
Attention is directed throughout to comparing the height aspect of head and the body in the pictures, rather than on their global size. The blocks and the rods on the side of the picture of the Jelly sweets point the attention of the children to the number aspect, but full use of number is not expected at this stage.

Episode 2: Deciding which of the other shapes is of which type.

Carrying on with the story of the sweet factory the teacher tells the children that each type can come in different sizes, so the the Jelly Daddy can be smaller than the Jelly baby. *'How can that be possible?'*

Children look at nine pictures and decide for each whether it is for a jelly baby, jelly child (Jelly boy, Jelly girl) or jelly daddy. None of the pictures are of the same size as the three given earlier, nor have they the blocks or rods to help.

To decide the type the pairs of children must find ways of comparing the head and the body using strips that can be folded or torn, and/or pencil marks using the head as a measure.. They must give reasons for their decisions. That may lead to discussion of ratios, recorded in some ways, as well as of fractions.



Episode 3: Enlarging the scale; A Jelly Giant
The teacher tells the class the manufactures wanted to have a Jelly Giant as well. She asks, *'What might you expect about the size of his head, compared with his body?'* For an answer of the kind: '*LARGE body*' the teacher presses for a comparison with the the head, rather than just the size, e.g. *suppose the whole sweet is of this small size, how can the body be large?* Then she can direct the children *to decide the size of the HEAD and Body of the Giant, measure up its side, and then draw it*. To make it more challenging the children are not to tell the scale so that others can guess or work it out. (all this work on comparative sizes is done on the basis of *lengths*.)
The children either use their card strips or maybe the blue and red cubes or lines from their Daddy picture to measure the head and body, and then draw it. Then the rest of the class will have to guess what numbers they have chosen. The teacher watches them as they try to draw, and decides on two that are clear drawings but at the same time different scales. These are then pinned up to the whiteboard, and the children are asked what sizes (number of head units) the children who drew the bodies used. She asks them for reasons and uses whatever measuring instrument they suggest to check their guesses/choices.

How far to go with the activity?
At each stage the teacher decided, according to the range of ability in the class, and the volunteering of ideas, how far to go in formalising the writing of the ratios. The emphasis in all cases is on ways of working: How useful were the paper strips, the pictures of blocks or sticks? How useful is using numbers?

With older pupils and the more able, the whole activity described above can take half the time that it took us with the Y2 class we trialled it in. Possible extensions include:

- Giving pupils heads alone or bodies alone in different sizes, with the label of what they are, i.e. Jelly Baby, Jelly Daddy etc, and asking them to complete it, using rulers or strips of paper.
- Suggesting that the sweet factory is thinking of adding to some jelly sweets small chocolate buttons somewhere half-way down the body. How can they find that position? What about if the button is half-way from top of head to bottom of feet? What is the difference between the two positions?
- Writing the ratios for each of the type of jelly sweet, how to write these as fractions.
- Then, for the still more able, how can they explain the difference between a ratio and a fraction. Hopefully some interesting spontaneous language descriptions can be aired to the effect that ratios are usually used when separate things are compared , while a fraction is part of the whole. In both cases there is a unit that is used for both parts. The potential for confusion can then be explored, since a fraction is really a special ratio kind of ratio.

We ourselves have not tried this activity with children older than 7. Perhaps teachers who try can report how it goes.

Kings College, London

[1] Butterworth, B, (1999). *The Mathematical Brain*. London: Macmillan.. Also Dehaene, S, (1997). *The Number Sense*. London: Allen Lane, and Dehaene, S, Dehaene-Lambertz, G., & Cohen, L..(1998). Abstract representations of numbers in the animal and human brain. *Trends in Neuroscience*, 21, 355-361.

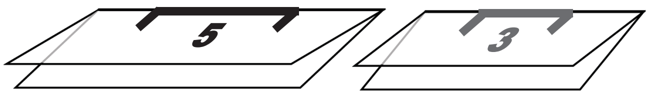
[2] *Realising the Cognitive Potential of Children 5 to 7 with a Mathematics focus* (2001/2004). Research project funded by the Economic and Social Research Council at *King's College*.

Our endangered environment
Ancient woodland has formed part of our landscape since the Ice Age. Sadly, it now covers less than 2% of the UK. …Over 40 species associated with woodland died out in the UK last century. … At the moment development plans threaten over 300 ancient woodlands Plans to destroy 160 acres of ancient woodland in order to build a new runway at Stansted Airport.could be prevented . In the last six years alone [The Woodland Trust has] planted 3 million native trees and protected hundreds of woods.
Leaflet, The Woodland Trust, September 2004.

From Simple Beginnings

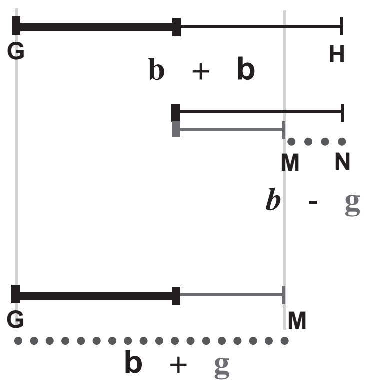
Many ways of making 8 from 5 and 3

It all began with the two segments of length 5 units and 3 units: the black length and the grey length - b and g - the beginnings of algebra.



Gerry (Gershon) Rosen now shows that the algebraic notation, in telling the story of how the segments have been manipulated, loses its bewildering abstraction.

It has already been agreed that
2 is the difference between b+b (10 units of length)
and b+g (8 units of length)
Therefore
8 must be the difference between 10 and 2 in one of its many forms.
So, the question now is: how do we draw it?
Remember the 2-unit segment has to be constructed. Which 2 unit segment construction would be useful to use?
After a great deal of thought and trial with different constructions of the 2-unit segment Irad proposed the following diagram.
First he drew the 10 unit segment ,from left to right.



Then he constructed 2units by drawing,
from right to left, the 5 unit segment
followed, from left to right,.by the 3unit segment.
In other words, he subtracted the difference between 5 and 3 from 10.
The diagram was problematic because it was difficult to "see" the 8 unit segment (GM) in the combination of the 10 unit segment (GH) and the 2 unit segment (MN). Irad solved the problem by drawing the two vertical lines shown

Irad demonstrated the identity

$$(5 + 5) – (5 – 3) = (5 + 3)$$

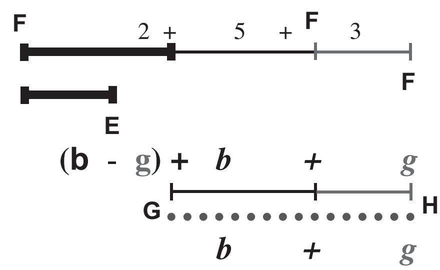
or in general

$$(b + b) – (b – g) = (b + g)$$

In effect he demonstrated the algebraic rules for eliminating brackets which we would write in the following manner:

$$(5 + 5) – (5 – 3) = 5 + 5 – 5 + 3$$
$$= 5 + 3$$

Leah used a different construction of the 2-unit segment. She started with EF of length 10 units. (5 + 5). She then increased the segment on the right using the 3-unit segment, making a 13-unit segment. Then she found the difference between the 13 and another 3 by lining both segments up on their left hand side. This in effect has moved EF, the 10 unit segment, 3 units to the right and is made up of the sum of three segments:
a 2 unit segment, a 5 unit segment and a 3 unit segment.
So now –
by adding the three units to one end of EF
and then removing it from the other –
now remove 2 units from the 10 units
leaving 8 units.



Leah in effect added zero in one of its many forms:. expressed algebraically:

$$5 + 5 + 0 = 5 + 5 + (3 - 3)$$
$$= 5 + 5 + 3 - 3$$
$$= 5 - 3 + 5 + 3$$
$$= 2 + 8$$

This enables us to remove 2 to achieve 8.

In conclusion

If we approach the learning of mathematics by teaching a multitude of rules (algorithms) to be applied to a multitude of mathematical situations , how do our pupils find the thread that links the different branches of the mathematics being presented?. It is often difficult to see a connection between one learnt chapter and the next. In this article I have tried to demonstrate a connection between various branches in the development of algebra without resort to learned algorithms. It is important to note that all this development took place through the manipulation of the two segments from the worksheet in the first section of this article. The worksheet was first used with 9th graders (15-16 year olds) who have "failed!?" in practically every stage of their mathematical education particularly in algebra. From the construction stage we moved onto the description of the processes using algebraic symbols and from there to develop the rules for algebraic manipulation but without a methods of constructing the solutions visually. Making use of several senses naturally gives rise to the use of algebraic symbols.

Western Galilee Regional High School, Israel

Less is Better!

Arno Rabinowitz argues, intriguingly, that the paucity of research into dyscalculia is an advantage for those of us who teach mathematics. It is easier to find useful information and we are not bombarded by the excessive choice and complexity which is characteristic of what is written about dyslexia.

This article starts out with Dyslexia, but be patient: Dyscalculia comes into its own long before the end.

The British Dyslexia Society suggests that about 4% of children are severely dyslexic and that a further 6% are mildly affected. Its website gives figures for dyscalculia as at between 3% and 6% of the population. There is reasonable agreement about these figures in the literature but many teachers are dismayed that dyscalculia has never received as much attention nor been the focus of as much research as dyslexia. They shouldn't be. The overwhelming growth of the literature about dyslexia has not been an unalloyed advantage. Too much choice makes life difficult, while less choice allows one to see the field more clearly. The more research there is, the more complex it becomes and the more difficult to apply.

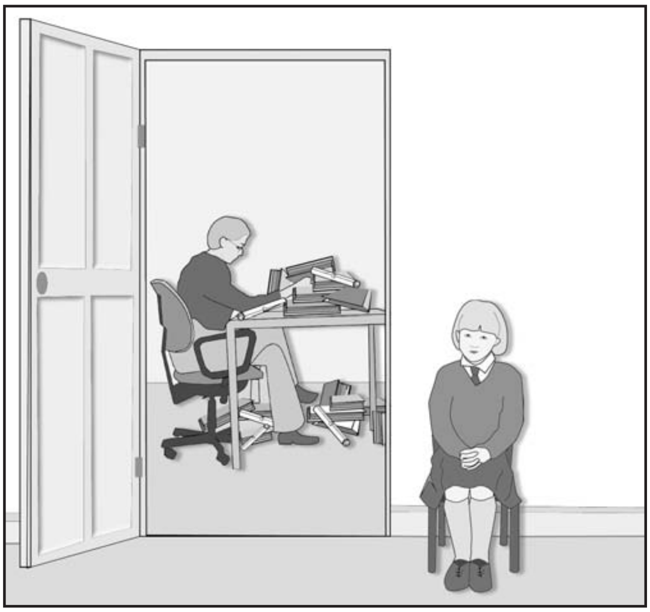
Lets start by looking at children in the simplest way and not to be confused by complexity

Most research has to do with academic topics and this often leaves little room for research into innovative ways of helping dyslexic children.

The super-power of disabilities

Dyslexia - or "word blindness" in its earliest manifestations – became part of pedagogic consciousness in 1877 and almost immediately interest in it grew at a tremendous rate. When Adolph Kussmaul, a German physician, transposed the neurological term "alexia" into "word blindness" there were probably more people who could not read and write than there were who could. The arrival of a label for those who found learning to read difficult quickly spawned an immense and overwhelming new industry. New research, publications and teaching material bounded into life and then expanded exponentially with extraordinary energy. It was as if not being able to read was the most significant handicap ever. In 1968, in the United States alone, there were nearly 50,000 books and 4000 research articles on the "..various important dimensions of normal as well as impeded reading processes." [1] Now it is probably almost impossible to catalogue all the scholarly works and research papers that exist about dyslexia.

In a relatively short time Dyslexia became the super-power of disabilities, the generator of untold research, publication, provision and litigation. Many learning difficulties were described as sub-groups of dyslexia. Dyscalculia is even described by some as "mathematical dyslexia". Unfortunately, the more that was published, the more that was "known", the more complex and dense became the literature. As complexity increased, the ability to help children in many instances actually diminished rather than improved. Increased choice and knowledge do not generate proportionate benefits for children and those who teach them. As research into and publication of material about dyslexia flourished so it became more complex and less easy to understand. Complexity increased until it seemed as though clarity had gone out of fashion. The research topics of dyslexia became more abstruse. Topics offered – none of much immediate use to teachers – embrace a fantastically wide field: the motor development of the families of dyslexics[2] , dyslexia and prematurity[3], as well as hypertension.[4] They make interesting reading, but does the range and variety help teachers? There is certainly a fantastic range of material and much to choose from, but is that really good?



Helen Webb, the UK managing director of lastminute.com. recently pointed that people just stop buying products because of

- pressures in everyday life (75%) (teachers know all about these!)
- having too much choice (72%)
- over-complex products (53%)

and Barry Schwartz in his newest book *"The Paradox of Choice:Why More is Less"*[5] echoes this. Overmuch choice, he says:

- increases the burden of gathering information to make a wise decision
- increases the likelihood that people will regret the decisions they make
- increases the feeling of missed opportunity and later regret about decisions
- increases the chances that people will blame themselves when their choices fail to live up to expectations

In March 2004 a simple Google search for "dyslexia" yielded 677,000 results written in English. The same for "dyscalculia" returned only 17,900. Blackwells of Oxford list 210 current books in stock about dyslexia and 6 for dyscalculia. Less is written about dyscalculia than dyslexia and this relative paucity is actually an advantage rather than a disadvantage.

There is less to choose from and choosing what to read and how to use what it teaches is much simpler and likely to be more effective. What does exist is easier to find, less obscure and usually more easy to understand. And there is yet another advantage: because of this less complicated field one is freer to think in clearer and more straightforward terms about what it is that one is attempting to do.

We should not expect the same from all

In the introduction to his wonderful book *"The Mathematical Brain"* Brian Butterworth[6] suggests that we all have the basic, universal ability to manage number and that this exists in all societies, albeit in different ways. He also suggests that the "….differences in the level of adult performance (*in mathematics*)[7] will depend on experience and education." …and that, even taking those factors into account, we should not expect everyone to be able to achieve in mathematics to the same level. That, he suggests, would be as illogical as "…..requiring everyone to show the same sense of colour in the colour co-ordination of their clothes, or the way they decorate and furnish their homes, or maintaining that we should all be equally good at putting words together to create narratives or poetry." (p.9) because most of us can see colour or talk normally. There is much, much more of interest about mathematical thinking in Butterworth's book, but his simple introduction is itself a great help.

This simply put proposition - that failure in mathematics depends on experience and education - has the benefit of the straightforwardness demanded by the principle of Occam's razor. This is a logical principle attributed to the mediaeval philosopher William of Occam and states that one should not make any more assumptions than the minimum needed.

Occam's razor helps "shave off" concepts, variables or constructs that are not really needed to explain whatever phenomenon is being examined. Reasoning becomes easier and there is less chance of introducing inconsistencies, ambiguities and redundancies.

Accepting the simple "shaved" proposition of Butterworth's introduction means that life does suddenly become much easier. You need simply to know clearly what there is in experience that is essential to learning mathematics and what there is in teaching that is essential. Experience incorporates knowledge, understanding and familiarity: if you know the knowledge, understanding and familiarity that is necessary for learning in mathematics and which of these a child possesses, then classroom diagnosis – without in any way denying the worth of good diagnostic tests like Butterworth's[8] - becomes much more possible….and successful treatment most often follows successful diagnosis.

Clearly there will be some children who find learning mathematics difficult because of a physical difficulty like visual or hearing disabilities. Some will have hidden problems to do with brain function such as those recently described by Molko[9]. He found that that people with dyscalculia that he tested had abnormal pulses of activity in a brain furrow called the right intraparietal sulcus. But most will have none of these. They will have weaknesses in essential basic skills or, equally importantly, will have been poorly or incompletely taught at an earlier stage.

Equals readers already know which basic skills a child needs for the learning of mathematics. They are:

- sound early language development and a reasonable age-appropriate vocabulary
- properly developed understanding of laterality, direction and left/right concepts
- a good sense of sequence, short term verbal and visual memory
- reasonable working memory for mental arithmetic and so that the sequence of operations can be retained
- the ability to understand and express abstract ideas

and they will know what needs to have been taught if success is to continue:

- how to count
- how to read and write numbers and use them in sets
- what number magnitudes represent
- facts of number bonding
- the ideas behind arithmetic operations
- ordering by size
- the meaning of operating symbols +, x, = etc.

If something is missing from either set then the child will have difficulty in learning mathematics. If any of the basic skills are missing or poor then learning will not happen efficiently if at all. If essential teaching has been missed then children will simply stop learning when the processes become too difficult or fail to make sense. Careful classroom diagnosis based on such simple schemata is both vital and usually successful. It doesn't have to be as complex or as profound as it can sometimes seem to be in dyslexia.

None of this is to minimise the complexities which underlie mathematical thinking and the need for mathematical competence. Butterworth thinks that he processes "……about 1,000 numbers an hour, about 16,000 numbers per waking day, nearly 6 million in a year." We all need to use numbers all the time. Not being able to do so is a very great handicap and, in employment terms, a greater one than not being able to read efficiently. However, if we are to begin to attack the task successfully, we need to start by looking at children in the simplest and most direct way and not to be confused by science and complexity. We can help children if we think clearly and are not too blinded by science or waylaid by the temptation to sound serious and be complex. Don't mistake what I say. Research is a great and wonderful and necessary thing but so is success with children and the two are not necessarily always interrelated.

[1] Klasen E (1972) : *The Syndrome of Specific Dyslexia*. University Park Press, Baltimore.

[2] Children with a family history of dyslexia have slow fine and gross motor development in the first two years. *Developmental Medicine and Child Neurology*, November 2002

[3] Learning Disabilities present in over 65% of normal IQ children who were of very low birth weight or premature babies. *Archives of Paediatrics and Adolescent Medicine*. June 2002.

[4] Dyslexia may protect against high blood pressure. *Archives of Disease in Childhood*. February 2002

[5] to be published in June 2004. Ecco/Harper Collins, London

[6] Butterworth, Brian (1998) :*The Mathematical Brain*. Macmillan, London.

[7] author's insertion

[8] Butterworth Brian (2003): *Dyscalculia Screener.* NFER Nelson

[9] Molko N et al (2003) *Functional and structural alterations of the intraparietal sulcus in a developmental dyscalculia of genetic origin*. Neuron. 40 (847-858)

Croydon, Surrey

RECTANGLES: area and perimeter

Questions and answers

Copy board onto OHT, or onto interactive white board, or enlarge to A3 for class display.
Use it for an oral/mental starter to address objectives from year 4 (calculate the perimeter and area of rectangles) to year 7 (know and use the formula for the area of a rectangle)

Some questions (and answers):

about area:

What is the area of the rectangle at, say 2A? Which rectangle has an area of, say 24 cm^2?

Table of areas

	1	2	3	4
A	24cm^2	35cm^2	18cm^2	56cm^2
B	180cm^2	42cm^2	36cm^2	24cm^2
C	70cm^2	33cm^2	88cm^2	120cm^2

Which rectangles have areas of less/more than 100 cm^2?

about perimeter:

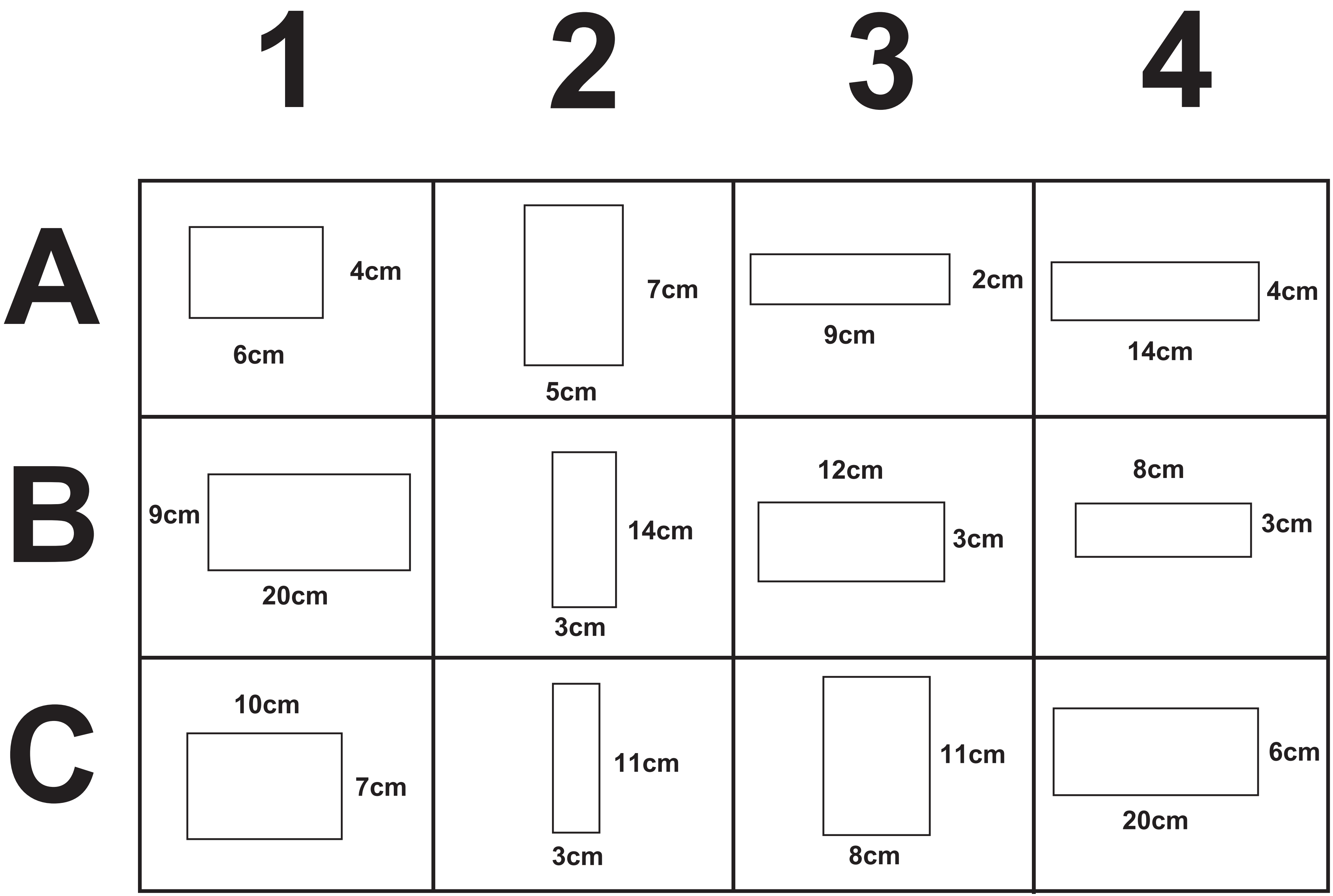
What is a quick way of working out the perimeter of a rectangle? What are the perimeters?
Can you find any rectangles with perimeters of the same length?
Which rectangles have a perimeter of 22cm?

Table of perimeters

	1	2	3	4
A	20cm	24cm	22cm	36cm
B	58cm	34cm	30cm	22cm
C	34cm	24cm	38cm	52cm

Which rectangle has the smallest area/perimeter? (3A/1A) Why isn't it the same rectangle which has the smallest area and perimeter?

Board (*drawings* ***not*** *to scale*)



The Excitement of Working Together

Arbour Vale teachers in working together discover how teaching and learning can be enhanced

Arbour Vale School is a special school in Slough catering for pupils aged 3 to 18 with the full range of special needs. The school is working on developing much closer curriculum links between the SLD and MLD departments. The article describes the early phases of this work in relation to mathematics in key stage 3.

Following an internal audit of the mathematics provision it was clear that there were great differences in curriculum coverage, not just between the MLD and SLD departments, but also within the departments. The planning tended to be one person's work, which was then shared and agreed with the other teachers and some adaptations made. It was felt that this was not as collaborative a process as it could have been; but collaborative planning takes time and this is a very precious commodity.

As a pilot project, fully supported by senior management, which was crucial to the successful outcome, it was decided to focus on an area of mathematics (handling data). After some initial planning discussions the head of the department planned a lead lesson with the aim of demonstrating high quality teaching to other staff in the introduction of a topic. The lesson would be taught to all the pupils in a large space in the school with all the other adults supporting. It was the role of the lead teacher to ensure that everybody, including support assistants, teachers and other adults, knew what they were supposed to do during the session. Thus the head of department was to communicate a way of working, show a range of curriculum coverage of the topic, demonstrate differentiation and, most importantly, to generate lots of post-lesson discussion. The SLD and MLD departments decided in the first instance to teach the pupils within the departments separately but with teachers from other areas involved in the lessons.

Each lesson was very much 'an event' and a great deal of planning went into the work.

SLD lead lesson

The room was arranged into five sections each of which showed a different colour. The whole lesson was about colours.

The children were asked to go to the section of the room with their favourite colour. It was difficult to see which section had the most popular colour but it was very clear that one colour (yellow) was the least popular, because it was obvious that there were fewer people there.

The children were then organised, those in wheel chairs as well, into a 'people' bar graph and a discussion took place as to how this might help answer the question: 'What is the most popular colour?' Adults in the room ensured that everyone was lined up correctly so that this could be done. This generated a lot of really good talk.

Lined up as a pupil bar chart in the SLD lead lesson.

Following this the pupils were divided into different ability groups, very carefully differentiated to ensure that all were learning at their own level but around the same idea. Block graphs of various types were produced in each group but the range of support varied and the ways of achieving the outcomes were very different.

As a plenary to the lesson, each group shared the work they had done and held up examples of what they had produced. They were all very proud of what they had done.

The lesson lasted 90 minutes and the pupils were fully engaged the whole time. Some pupils still wanted to share their ideas when the others had gone out to break.

MLD lead lesson

There were 52 children in this group and 20 adults. After an initial mental arithmetic warm up the pupils were asked to undertake an investigation into the colour that occurs most frequently in a packet of Skittles.

Hard at work sorting the sweets and recording their results.

The pupils had to pick out 10 skittles and record the colours on a 10-section pie chart. The pupils were not grouped by ability and the adults in the room were asked to support and ask questions about what the pupils were doing:

- If I choose another 10 will the red still be the most popular colour?
- How can I group these Skittles?
- Are there any purple skittles in the packet?
- How should we colour in the pie chart?
- How could we use this pie chart if we took out 20 Skittles or would we need two pie charts?

Pupils working on recording their experiments on a pie chart.

Some pupils grouped together on the pie chart some did not - an interesting discussion for the plenary.

As a result of these probing questions pupils were beginning to group according to colour and starting to get intuitive ideas that a pie chart shows proportions. Some of the pupils were beginning to compare using language like half, thirds, quarter etc.

The plenary to the lesson, conducted with the whole class, enabled this discussion to take place with everyone, including the teachers, contributing.

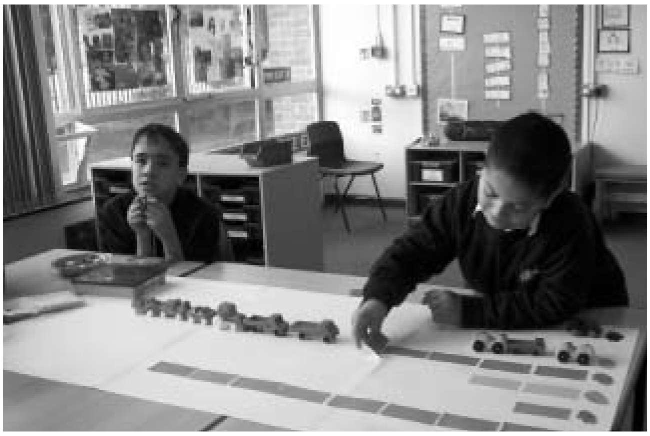
What happened next?

Following the lessons there was an air of excitement around the school. SLD and MLD teachers talked together and those who had not been present were keen to know what had happened and sought opportunities to see other lessons. Within the MLD and SLD departments there was lots of discussion and sharing of ideas about how to follow up the lesson with their groups. There were 5 follow-up lessons in total.

Teachers were able to see what to do next because of the experience of seeing the lead lessons. Teachers could target pupils of middle ability with questions and work which they had seen more able pupils accomplishing in the lead lesson. The continuity and progression evident in both lessons had set an agenda for follow-up work. The teachers and the pupils had a clear idea of where they were going.

The teachers of high ability pupils had higher expectations of them because they could see the sorts of achievement possible with the right activity and the right stimulus, be it a question, a game or an investigation. For example, in a lesson where pupils had to make random picks out of a bag containing different proportions of multilink and record the outcomes onto a spreadsheet which produced pie charts and bar charts, some pupils were able to interpret the graphs to correctly state how many of each colour were in the bag without looking in the bag.

These were quite sophisticated levels of interpretation and understanding of probability which might not have been possible without the work on the Skittles investigation in the lead lesson and the probing questions used in the lesson.



Sorting the colours of toy cars.

Throughout the five follow-up lessons, teachers were encouraged to visit classes across the two departments.

At the end of the project a review meeting was held in which everyone had the opportunity to discuss the benefits or otherwise of what had taken place.
Key areas of benefit identified within this project were:

- It demonstrated the benefits of collaborative enquiry among teachers and support assistants as a vehicle for real school improvement within the school.
- It improved the quality of planning and teaching and hence learning.
- It was an enjoyable way of working!

There was general agreement that the professional development was an incredibly positive experience and everyone is keen to explore other opportunities, not just in mathematics, to develop this further. Interestingly, no one went out of school on a course, very little supply cover was needed and everyone, support assistants (who were fully involved) and teachers, had really enjoyed the learning experience.

Early Steps in Number Multiplication and Division

This time **Stewart Fowlie** offers a series of activities on meaningful multiplication and division to help to avoid pitfalls that make lasting confusion in the minds of youngsters. All is based on handling **NUMBER as a QUANTITY**

Just as the teacher should introduce addition and subtraction through situations involving their use, so I suggest multiplication and division should be handled. The first situation might be **dealing cards to players in a game**. The first column in the three column table would be the number of cards each player is to get (size of hand), the second column, the number of players and the third, the total number of cards needed (size of pack).

Number of cards each player has	Number of players	Total number of cards
4	3	?
?	4	20
3	?	15

The first exercise would give an entry in each of the first two columns, and each child would be given more than enough cards **to make up the hand** specified to the given number of players.

Consider the task of finding how many cards are needed to give 4 cards to each of 3 players. Notice that he should make up 3 little piles of 4 cards initially rather than dealing out one card, then a second, a third and a fourth to 3 piles, because that is really working out 3 cards 4 times rather than 4 cards 3 times. (This of course illustrates the commutative principle for multiplication, but that should be discovered later.) The number of players and the size of the hands should vary up to perhaps 5. The children are then not learning about multiplying by a specific number, but about multiplying as a concept. What the answer means should always precede knowing what the answer is.

Consider working out the entry in the first column if that in the second column is 4 (number of players) and that in the third is 20 (size of pack). To give one to each player needs 4 cards, so one finds one can give 5 cards to each player. If on the other hand the entry in the first column (size of the hands) is given as say 3, and that in the third (size of pack) is, say, 15, one can see that the number of players is 5.

What the answer means should always precede knowing what the answer is.

The thought process is different in each case. In the first case one is dividing a number of cards by a number of players, and getting a number; in the second, one is dividing a number of cards (the total) by a number of cards (in each hand) and getting a number of players. The first case leads to the concept of division as repeated subtraction (the reverse of multiplication as repeated addition) the second leads to a fraction (possibly an improper one). It is not surprising that fractions cause difficulties when the way division is normally done (repeated subtraction) has little apparent connection with fractions. If all this is reduced to 3 x 4 = 12 so 12 ÷ 4 = 3 or 12 ÷ 3 = 4, how much understanding is lost! Indeed it might be better at first to talk about division when one is dividing a quantity by a (counting) number, but use a different word, perhaps fractioning, when one is dividing one quantity by another quantity. To most children, dividing by boys seems just as incomprehensible as multiplying by boys! At this stage children should consider the sharing of, say, 8 cards between 3 players as impossible, and certainly not give an answer 2 r 2 .

Starting with the product
Just as with subtracting, children will see the situation as different if the entry in the first column has to be divided by the entry in the second column to get the answer in the third column, and situations of this type should be turned into exercises, and of course those based on numbers of different things. Here are a few suggestions:

At this stage children should consider the sharing of, say, 8 cards between 3 players as impossible.

- Total number of legs of a group of children, dogs, insects, or spiders. (if some insects have 17 legs between them there must be 3 insects, one having a leg missing!)
- Total cost of a few sweets costing an exact number of pence each.
- Total cost of cinema tickets for a family (each ticket costing an exact number of pounds) .
- Total length of a number of (match) sticks placed end to end .
- Perimeter of triangle, square, pentagon, hexagon, made of match or other sticks .
- Weight of bag of so many pieces of fruit, each weighing 10g, 20g

Attention to language in multiplication problems
When multiplication is first introduced, it is usually by saying that, for example, 3 + 3 is just 2 times 3, 4 + 4 is 2 times 4 and so on. From work with addition, the children know that 3 + 3 is 6, 4 + 4 is 8 leading to a list of results 2 times 1 = 2, 2 times 2 = 4 … 2 times 10 = 20. More mysteriously this may be written 1 x 2 = 2, 2 x 2 = 4, … 10 x 2=20, which may be easier to write but harder to say as "Ten multiplied by two = twenty". Children will come to know that 2 x 10 comes to the same thing as 10 x 2, and they could visualise the first as 10 lots of 2 (meaning "the 2 repeated or multiplied 10 times") second as 2 lots of 10 (the 10 repeated twice). But in real life they should recognise the difference. If you are asked to buy 6 25p stamps, and come back with 25 6p stamps your Mum will not be best pleased, even though they cost the same.

In life, the multiplicands are of different nature
What this obscures is the rather important idea that whereas we can add 2 boys and 3 girls to get 5 children we do not give a meaning to 2 boys times 3 girls, or 3 girls multiplied by 2 boys. We do give a meaning to 3 boys multiplied by 2, or 2 times 3 boys. It means 2 lots of 3 boys. It is meaningful to write this as 2 times 3 boys = 6 boys, or 3 boys x 2 = 6 boys (each of which means precisely the same), but what is nonsense is to write either 2 boys times 3 or 2 x 3 boys (sometimes one hears "you times the boys by 3", but that should be written as 3 times 2 boys or as 2 boys x 3) You can multiply by 3 but not by 3 boys.

When adding everyone quickly realises that 3 boys + 2 girls is the same as 2 girls + 3 boys, but if they ever thought about it they would realise that it is not the same as 2 boys + 3 girls. In both subtraction and division it is vital that 3 - 2 is not confused with 2 - 3 nor 6 ÷ 2 with 2 ÷ 6: the first number is acted on by the second, so it is better to write 3 x 2 rather than 2 times 3 (and if you refer to them at all to refer to multiplication tables rather than times tables). There would be no confusion if 3 x 2 were read as "three by two" and 3 ÷ 2 as "three over two" .

There is something wrong with our teaching if we use a real situation to clarify addition at the age of 5 or 6, but by age 12 number work is good, but so-called problem solving (or applying arithmetic to real situations) is found very difficult. Perhaps things would be better if a number were always thought of as a number of things. Perhaps there would develop greater understanding of number as an approximation to an appropriate accuracy of a measure rather than only as a measure of how many.

Related to this approach would be to have letters in algebra standing for quantities rather than for numbers, as is the practice in coordinate geometry and in physics. The turning of a problem in algebra into an equation often starts by writing "let the length be x cm."

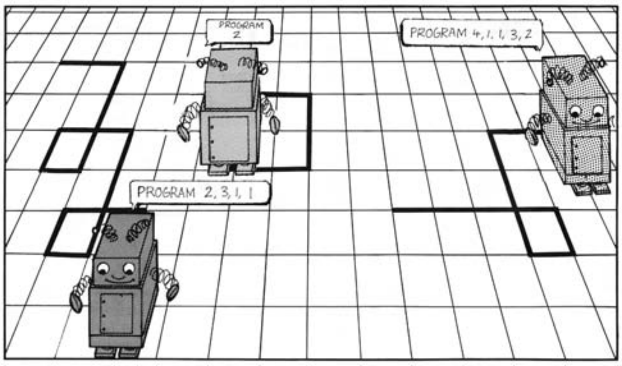
The equation then obtained is about the number x. Is it not easier to understand 2x + 7cm = 11cm where x is the length of something than to understand 2x + 7 = 11 where x cm is the length of something? Surely it is then even more baffling for the child to be invited to think of its sides as weights which balance .

I have a book originally published in 1959 called "A Concrete Approach to Abstract Algebra." Perhaps I should acknowledge indebtedness to its author W. W. Sawyer for the idea of the above, calling it "A Concrete Approach to Abstract Arithmetic". ***Make it real!***

Edinburgh

Loop the Loop

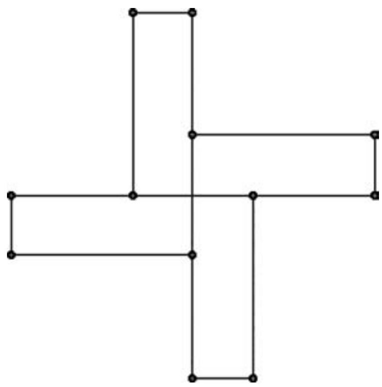
Emma Saunders gives an example of a mathematical activity which could be adapted for any age or ability group. It could also be used for an early lesson with LOGO to introduce the REPEAT command.



I was approached to teach a one off maths lesson for eight primary schools in our local area. The schools were asked to send four of their most able mathematicians from years 5 and 6 for this enrichment afternoon. We decided on an investigation based on spirals, as it lent itself to predicting and generalising. The children were also able to suggest extensions to the investigation by asking a variety of 'what if' type questions.

We started off by taking any three numbers e.g. (3, 1, 4) and I demonstrated by walking forward three strides. I then turned 90 degrees right, moved one stride forward, turned 90 degrees right again and then moved four strides forward. I repeated this programme until I was back at my starting point. I then demonstrated how to draw this out using squared paper.
The first question we decided to investigate was *What if we changed the order of the numbers?* We discovered that some spirals were mirror images of each other: (1, 5, 3) and (5, 1, 3), whereas other combinations produced the same result: (1, 5, 3) and (3, 1, 5). Many of the children began to notice that if the numbers were in the same order then the spiral would look the same.

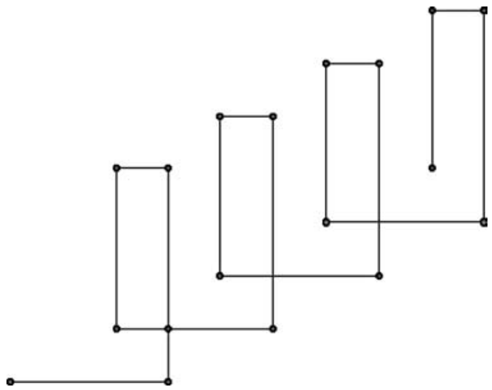
We then chose to investigate *What if two of the numbers are the same?* For example
(3, 1, 3) and (1, 3, 1). Again, the children soon began to predict that some combinations would produce crosses and others produce squares. They also realised that there was a connection between the middle number in the programme and the area of the middle square of the cross.



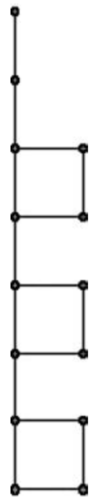
3,1,4

Next, we investigated *What if we tried consecutive numbers?* such as (1, 2, 3). The children were able to draw comparisons between the size of the overlapping rectangles produced and the numbers in the programme.

By this time, the children were very keen to pose their own questions, such as *What if there are four numbers in the programme?*



Using 4 numbers - 3,1,4,3



1,1,1,3

What if the numbers are repeated? (1, 3, 1, 3) *What if three numbers out of four are the same?* (1, 1, 1, 3) They were encouraged to follow their own lines of enquiry.

I was particularly impressed with the enthusiasm with which they tackled the task. Mixed gender groups worked together equally as well as single gender groups. As the session progressed, the children were obviously learning from each other. Some children began to use correct mathematical vocabulary, having heard others using it appropriately. Others became much more methodical in their investigating after sharing ideas with their table group. Spurred on by other ideas, children who had been initially reticent, particularly when coming up with their own questions to investigate, began to suggest their own line of questioning.

Back in my own classroom I noticed that children who had been to this enrichment session were able to lead the others when carrying out a different investigation, particularly in the areas of suggesting what if questions to pose and working logically.

Music, Mathematics and Language.

Steve Grocott explores some of the connections between maths, music and language that go deep and wide. He suggests how music might help those who find the mathematical bit somewhat of a struggle.

"Music is a secret and unconscious
mathematical problem of the soul".
Gottfried Leibnitz (1664 - 1716)[1]

I invited my cellist friend to a primary school to share her wonderful playing with the children. This is a musician who can reduce hardened cynics to tears with the rich river of sound that she draws from the beautiful instrument. As the children sat waiting, I mentally rubbed my hands together anticipating the moment of connection. She picked up the instrument, looked at the children and said "Now, how many strings have I got on my cello?" This is clearly not what Leibnitz was talking about but we have all done it. Faced with a group of children we go into "Education mode" and the richness of what we have to communicate goes straight out of the window.

The connections between maths, music and language are deep and wide. They have to do with perceiving and making patterns. Research is demonstrating more and more how the brain is a pattern-perceiving and pattern-making thing. (We are born with a series of templates and set about finding their counterparts in the environment.[2]) Maths, music and language can all be seen as the fruits of this ability. They are not a sort of add on that develops when we acquire the power of abstract thought but are connected at a basic level to our ability to survive.

We are born, for instance, with a need to connect to other people and the basis of the skills to do it. One of the first imperatives is establishing rapport with our mother.

Interactions between babies and mothers have been analysed and show that from birth we have the capacity to converse in rhythm. This insight was gained when the scientist Colwyn Trevarthen teamed up with a musician who analysed recordings of the "conversations" between mothers and newly born babies. The recordings could be plotted on a computer music programme and the way the mother and baby made sounds to each other - taking turns, echoing, leaving regular gaps and making phrases - could be clearly seen to fit into a musical rhythmic schema.

Staying with the communication aspects of music for a moment, the extent to which musical elements contribute to language is beautifully demonstrated by the language of the Clangers in the children's series. There are no words but sounds that evoke the pitch, dynamics, form and timing of speech made with a swanee whistle. Oliver Postgate, the creator, did script the shows so the patterns are authentic. (In fact the script was edited by the BBC for swear words). We understand what the characters are saying without any words. Of course the context and action play a major part too. When Postgate went to a European convention on children's programmes he discovered that German delegates thought the Clangers were speaking German and French speakers that it was French, etc. This tells us something else about the way we perceive patterns. It is an active process. The brain takes in a few clues and makes sense out of them according to the patterns that are already there.

The connections between maths, music and language … have to do with perceiving and making patterns

"Where's the maths?" you may be asking by now. Let's look at an example. In my band we have a song that follows the pattern of "She'll be coming round the mountain". This can be expressed in beats :-

1 - 2 - 3 - 4 - / 1- Toot Toot 4 -
1 - 2 - 3 - 4 - / 1- Toot Toot 4 -
1 - 2 - 3 - 4 - / 1 - 2 - 3 - 4 -
1 - 2 - 3 - 4 - / 1- Toot Toot 4 -

We sing the song and then hand out a train whistle to the first child who expresses interest in joining us. In years of doing this we have never been let down by one of our child prodigies or the rest of the audience who cheerfully toot along in the right places. Children as young as two regularly absorb this pattern, never trying to play on the third line for instance, without any teaching at all. They seem to know it already. It is no accident that there are many songs that follow this pattern like "If you're happy and you know it clap your hands" and so on. To describe this pattern in terms of number requires a great deal more conscious brain work than communicating it in song but that is what we should start to do if we want to use musical activities to help with the learning of mathematics. We simply become aware of the connections and point them out to the children. Using this song we need to count the four lines of the song on our fingers as we sing. Then we can ask questions :

- Which line has no "toot" ?
- Which lines are the same and which different ?
- What does it sound like if we sing them in a different order ?
- Can we use blocks or other objects to make a score ?
- Will the score be the same if we make another sound for "toot" ?
- How many "toots" in each line - how many in one verse, or the whole song ? Then we can link this with learning the two times table.
- What other songs have this pattern?
- Let's make one up on the theme of animals.

A very good way in to dealing with the last question is to go back to the Clangers and imaginary language. If we make pretend language sounds to the rhythm and form of the song it makes a good basis for then finding words to fit the pattern. (Of course we might decide that the nonsense is good enough as it is, especially if our sounds to replace the toots are suitably amusing.)

When I was starting to think about writing this all my music sessions took on a mathematical tinge and I had to be careful not to lose sight of the task in hand by asking too many 'mathematical' questions. I would remind myself of the cello player and get things back into balance.

[1] Leibnitz quoted in *"The Wordsworth dictionary of musical Quotations"*
ISBN 1-85326-327-3

[2] See p 94 *"Human Givens"* - Joe Griffen and Ivan Tyrrell
ISBN 1 899398 26 0 Human Givens Publishing 2003
www.humangivens.com

London

Why Measure?

After a fantasy start, a review of Alex Hebra's *Measure for Measure: the story of Imperial, Metric and Other Units*[1] provides **Rachel Gibbons** with a frame for a brief historical exploration of some of our systems of measurement

Once upon a time a king wanted to give his queen a birthday present. He decided on a bed and paced out the length and the width. This is, in brief, the beginning of another book on measurement, *How Big is a Foot?*[2] Sadly, when this country "went metric" the book went out of print. Sad because it gave a vivid illustration of the need for standard measures. You see, the king gave the bed measurements to his little page who then paced them out for the carpenter. Of course you've guessed, the bed was too small. The tale was delightfully told and, although a fiction, it is essentially a part of the history of the evolution of standard measures over the centuries. It demonstrates a simple answer to the question, **why** should we measure? We very often measure, particularly lengths of objects, to answer the other question – **will this fit?** The king needed a bed fit for a queen, both in size and style. We need clothes that fit our bodies, furniture to fit into rooms, pitches into playing fields. Measurement activities in school should always be related to some other activity – constructing something or making something fit. It should also be put into a historical context. As children begin to use primitive methods of measuring they should appreciate that similar methods have been used in times past and learn something of how they have evolved into today's metric system, now used universally - although plenty of "convenience units" are still in everyday use in one circumstance or another.

In *Measure for Measure* Hebra gives some of the story of this evolution. His researches go back a very long way. He tells how round about 400,000 years ago the so called Peking man developed the ability to develop fire, and cites evidence of the use of flint and tinder in the young Palaeolithic period. Heat, he goes on to explain, is a form of energy, the kinetic energy of the molecules and atoms that compose matter. The lowering of a body's temperature, he continues, amounts to slowing down the oscillations of the atoms or molecules, "which at –273.15° C (absolute zero) come to a virtual standstill", making lower temperatures impossible. It took centuries for all this to be discovered because no measurements can be made without measuring equipment. Hebra therefore leaps onward in his story of the measurement of heat to the rudimentary thermometric devices of the early seventeenth century and to 1714 when Daniel G. Fahrenheit built calibrated mercury and alcohol thermometers and so on to Celsius and Kelvin whose scale "became in the International System of Weights and Measures as the Thermodynamic Temperature Scale".

In his chapter "Going to Great Lengths" Hebra starts with the Egyptians and the flooding of the Nile destroying all the boundaries between parcels of land belonging to different people. These boundaries had to be reconstructed when the floods subsided which resulted in the invention of the origins of geometry. For their surveying the Egyptians, like the king in our story above, used a comparison with part of the body to measure length - this time the distance from a man's elbow to the tip of his middle finger, the cubit. The cubit is also found, Hebra reminds us, in the early Bible stories, particularly the story of Noah's ark (Genesis 6:15). He notes that the ark was to be 300 cubits long, 50 cubits wide and 30 cubits high, "or roughly 450 by 75 by 45 feet – not much space in which to store two animals of every kind for forty days and nights!" The cubit was standardised at an early stage, Hebra tells us, when a $10\frac{1}{2}$ inch cubit rule was carved into a statue of the Sumerian king, Gudea who ruled the city of Lagash from 2197 to 2178 B.C. The cubit lived on, albeit defining a different length in each case, as a measure of length for the Hebrews, the Babylonians, the Greeks and the Romans.

Hebra discusses "convenience units" of measurement, telling how they arose and how succeeding generations were loathe to let them go when newer systems of units were introduced. He instances Eratosthenes of Kyrene's use, in his attempts to measure the earth's circumference, of "camel-days," commenting that the distance a caravan could cover per day was pretty uniform and well known. In our own time, Hebra writes, the Swiss guide when asked how far it is to the top of a mountain will say something like "five hours". We might add the measurement of lengths of journeys on London's underground by the number of stops on the way.

And perhaps even note that British Waterways have recently revamped a scheme for calculating distance in terms of 'lock/miles'. This is used to decide whether or not people who live on canal boats have moved a sufficient distance within a given period of time to not incur mooring fees, as they retain their permanently cruising licence.

Bill Bryson in *A Short History of Nearly Everything* tells the story of the French Academy of Science's Peruvian expedition of 1735 for the purpose of triangulating distances through the Andes.[3] "At that time", he writes, "people had lately become infected with a powerful desire to understand the Earth – to determine how old it was, and how massive, where it hung in space, and how it had come to be."

Such contexts can give children a view of measuring as not only a useful activity but an exciting one. It is possible to find plenty of excitement that we can add to lessons in measurement at all levels in outline histories of the measurement of length, turn, mass, weight, luminosity, etc., showing how they have finally all "gone metric".

1 Alex Hebra, *Measure for Measure: the Story of Imperial, Metric and other Units*, Baltimore: The Johns Hopkins University Press, 2003

2 Rolf Myler, *How Big is a Foot?* Wheaton.

3 Bill Bryson, *A Short History of Nearly Everything*, London: Black Swan, 2004.

Fulham, London

Correspondence:

email:R.P.Burn@exeter.ac.uk **18 April 2004**

Dear Editor

I was very interested in the article by John Searl and Sharmila Sivalingam on "Dyslexia and mathematics at university" in Equals Vol. 10, No. l . They had monitored a group of 9 dyslexic students out of a class of 400 taking mathematics courses as undergraduates.

The first finding of note was that this group was not distinguished statistically from a similar group of non-dyslexic peers on the assignments Algebra 1 and Calculus 1. There was a distinction on the assignments Algebra 2 and Calculus 2. The difficulties identified amongst dyslexic students were not uniform across the group, and I would not be surprised to find the actual difficulties identified occurring amongst non-dyslexic students. When the authors gave an overview of the difficulties, they used language which, I conjecture, might well have been appropriate for a random group of 9 students:

Many in this group appeared to be frustrated at not being able to cope well with undergraduate mathematics especially since they had little or no problems with mathematics in school.

In the final paragraph the authors describe the transition from school to university as a transition from a supportive to a hostile environment.

It would seem to be eminently worthwhile to monitor a random group similarly. If the same findings were to be established more generally, they might go some way to explaining why university mathematics departments have such a terrible record in fostering the nation's future school teachers of mathematics. All the good ideas proposed by Professor Adrian Smith to improve the teaching of mathematics in schools may be being undermined before the horses leave the stable.

Yours

Bob Burn

R P Burn

Gypsy Maths

Tracy Wogman (tracy@wogman.freeserve.co.uk) wrote to *Equals*:

> Do you have, or do you know where I can find out, the gypsy maths method of working out multiplication tables used to assist children with learning difficulties?
> I have searched the internet but to no avail!!

Our response, sadly, was not very helpful:

One member of the team did come up with the following reference but noted that it does not mention multiplication tables.

http://www.congress-consult.com/mes3/Papers/CharoulaFragiskos.doc

Another member of the team wrote:

> At Central Foundation I saw what I thought of as Bengali maths: girls using their finger segments for counting, so they have 15 in each open hand, thirty on the two, and 60 if they flip the hand over. They knew where the 10 is and where, say, 18 is located, and could count on. A kind of counting line to 60. Of course it provides for counting in 3s. Considering the ancient origins of the Gypsies in India....

When we wrote to apologise to Tracy for our failure and asked for more details she told us:

> a maths tutor originally taught me gypsy maths, about 6 years ago, when one of
> my sons was having difficulty learning his multiplication tables.
>
> To the best of my recollection finger-tips, that had been given a numerical value, were placed together forming an arch: fingers before the arch were 'tens' and the fingers behind the arch were 'units' which added together formed the answer.

Together with Tracy, we would be interested to find out more.

Can you help ?

Haiti

In 2000 Aristide took the presidency for the second time with 92% of the vote.

80% of the population endure poverty, malnutrition and disease.
It is highly overpopulated with approximately 8.3m people crammed into only 27,750 square kilometres.
Average earnings are just over £1.50 a day and life expectancy at birth is just 49 years for men and 51 years for women.
Nearly 50% of the population are unable to read.
Many people are malnourished, surviving on only 75% of the minimum required daily calorie intake.
The richest 1% of the population own half the country's wealth.

"Alan Parr" df08@dial.pipex.com   04 May 2004

The story about sharing a cake between seven reminded me of a similar experience.

Again, seven people, and again the decision to cut into eight portions. But being greedy, no-one was satisfied with this. So the eighth piece got itself cut into eighths, with the gleeful recognition that - at least in theory - the final eighth could be cut into eight more pieces, the eighth of which
....

Some powerful mathematics here, made easily accessible. The same result in conventional notation would terrify most pupils (and teachers) and look something like:

$1/7 = 1/8 + (1/8)^2 + (1/8)^3 + (1/8)^4 + ...$

Which, in turn, further reminds me that Michael Rosen's poem "If you don't put your shoes on" from, I think, "Quick , Let's Get Ot Of Here" has similar potential in communicating ideas about fractions and infinite series.

All good wishes,

Alan Parr

Reviews

Review   Jane Gabb

Working with Dyscalculia
Anne Henderson, Fil Came and Mel Brough
Learning Works
01672 512914
ISBN 0-9531-0552-0
www.learning-works.org.uk

Deep Progress in Mathematics: The Improving Attainment in Mathematics Project
Anne Watson, Els De Geest, Stephanie Prestage
University of Oxford, Department of Educational Studies
15, Norham Gardens, Oxford, OX2 6PY
ISBN 0-903535-68-8

Both of these books address the problems that low-attaining pupils have in accessing the mathematics curriculum; that is what they have in common. However, they approach this challenge from very different viewpoints: *Working with Dyscalculia* starts from the premise that Dyscalculia is a condition; Deep Progress is an account of a project where teachers were supported in exploring approaches to mathematics with low-attaining pupils.

Dyscalculia is becoming the new dyslexia - everyone is talking about it. In a series of recent visits to schools it was mentioned by teachers in many of those meetings; at similar meetings last year, it wasn't mentioned once. Is it helpful, or is it just another label which provides excuses for not addressing the needs of low-attaining students in our schools, and more widely in the adult population? Instead of saying 'I was never any good at maths' will people begin to say 'Of course, I can't do maths, I'm dyscalculic.''?

However, both books offer good practical suggestions for work in the classrooms.After giving an overview of dyscalculia, *Working with Dyscalculia* begins with a chapter entitled Removing Barriers to Learning.

The most interesting finding here is from some research (Askew, 2002) which identifies three belief orientations in teachers: Discovery, Transmission and Connectionist. Teachers with strong connectionist beliefs (attending to both what has been learned as well as what is to be taught) were highly effective, while the teachers with the other belief systems were only moderately effective. This seems to me to be vitally important and connects with all the research and practice on Assessment for Learning. However, it is merely mentioned in passing, the full reference is not given and it is not referred to explicitly again - perhaps the authors believe more in transmission than connectivity!
The focus is on the dyscalculic pupil - 'within-' and 'without- child factors' – with more about the individual than the context in which the individual is found.

The chapter on *Assessment* concentrates on finding out the strengths and weaknesses of individual learners and includes memory, organisation, reading and thinking and learning styles. At this point the book becomes much more useful and practical. This continues with the chapter on *Practical Strategies*, which is full of ideas and resources that will support teachers (and parents) in helping low-attaining pupils. At several points during the book there are pupil comments on How to teach and How not to teach; there is useful material here, but because it is not expanded in the text, it may be skipped – illustrations, perhaps even cartoons would have helped to make this stand out more.

I found *Deep Progress* an inspiring book; on the strength of it I have set up a group of teachers of low-attaining pupils in Windsor and Maidenhead. The starting point here is the belief that *all students can think hard about mathematics, and thus do better at mathematics*. The chapter about the students focuses on the context in which these pupils have been taught, even when it identifies 'within pupil' factors such as reading and writing difficulties and cognitive problems. It is very clear that the responsibility is the teacher's and the school's in terms of expectations and addressing the pupils' difficulties. Here, the research on connectionist teachers is not only mentioned and fully referenced, but informs much of the content of the book. [Askew, M., Brown, M., Rhodes, V., Johnson, D. and Wiliam, D.(1997) *Effective teachers of numeracy*. London: King's College.]

The teachers approached the teaching of their low-attaining pupils in different ways, but always with high expectations of what could be achieved. There are short chapters on different aspects of what the teachers did, including:

- Establishing work habits
- Generating concentration and participation
- Working on memory
- Being explicit about connections and differences
- Typical task types
- Structures of lessons

In each of these there are practical suggestions and ideas, which will be helpful to all teachers – on the premise that good teaching for low-attaining pupils will be good teaching for all pupils. There are so many that it is difficult to pick out examples. There is an emphasis on making learning and connections explicit and on giving pupils strategies to enable them to help themselves.

This is a short book, easy to read, and one which I will refer to and revisit for many years. Needless to say, dyscalculia is not mentioned once! An index would have been helpful to ease the finding of half-remembered ideas and examples.

Royal Borough of Windsor and Maidenhead

Million Children Campaign

It is estimated that around half a million households are overcrowded. … Over 1 million children in Britain suffer in bad housing.. … We want 100,000 to support the Million Children Campaign,

ShelterUpdate, Shelter, September 2004.

Who has Weapons of Mass Destruction?

...the revealing observation of Tony [Blair}'s that 'the United States has as many weapons as the next nine biggest nations…

Robin Cook, *The Point of Departure*, London: Simon & Schuster, 2004.

Healthy eating?

Ingredients: Milk chocolate (25%) (Milk, Sugar, Cocoa mass, Cocoa butter, Vegetable fat, Emulsifiers (E442, E476) Flavourings), Oats (19%), Invert sugar syrup, Bran flakes (10%) (Cracked wheat, Wheat bran, Sugar, Malt extract, Salt), Glucose syrup, crispy cereal (7%) (Wheat flour, Rice flour, Sugar, Dextrose, Salt), Hydrogenated vegetable oil, Sweetened dried cranberry pieces (5%) (Cranberries, sugar, Glycerine, Citric acid, Vegetable oil), Honey (2%), Humectant (Glycerine), Salt, Molasses, Emulsifiers (E471, Soya lechitin), Flavouring. MAY CONTAIN SESAME SEEDS AND TRACES OF PEANUT, NUT AND EGG.

From label on cereal bar.