# Thinking about Reasoning in Upper Primary

(in the light of end of Primary Statutory Assessments….)

Jo Lees : jlees6@aol.com

Before we start……Lots of ideas taken from:
Nrich; NCETM; MNP; MA; ATM (Thinkers book); TTS; STA; Jo Boaler and Mike Askew

Three aims for the National Curriculum for mathematics in England:

- Fluency
- Problem Solving
- **Reasoning**

- Reasoning is fundamental to knowing and doing mathematics
- Some would call it systematic thinking.
- Reasoning enables us to make use of all our other mathematical skills
- Reasoning could be thought of as the 'glue' which helps mathematics makes sense or a chain that links mathematics together so that one part can be used to make sense of another part.

*Jennie Pennant: Nrich: 'Reasoning; Identifying Opportunities'*

# Consider five key questions

If you are only allowed to use five questions in your maths lessons, what would they be?

## Discuss

# Thinking time !

As you engage in this session, listen for any questions

Make  a note of any that encourage you to think (differently)

Can you give me an example of:
a pair of numbers that differ by 2;
and another;
and another…..

What if I change **differ** to **sum**?

Can you give me an example of:
a pair of numbers whose sum is 2;
and another;
and another….

Which is harder and which is easier?

*From 'Thinkers', ATM*

What if I change **sum** to **product**?

Can you give me an example of:
a pair of numbers whose product is 2;
and another;
and another….

Which is harder and which is easier?

What if I change **product** to **quotient**?

Can you give me an example of:
a pair of numbers whose quotient is 2;
and another;
and another….

What if I change **2** to **3** ?

Harder or easier?

*From 'Thinkers', ATM*

If I know that 3.1+ 6.9=10 then what else do I know?

Can you give me an easy (simple) example of a calculation with an answer of 10 ? **Write it down**

Can you give me a hard (complicated) example of a calculation with an answer of 10 ? **Write it down**

**What is the same and what is different about the two calculations?**

56 x 3.75 = 210
210 ÷ 56 = 3.75
210 ÷ 3.75 = 56

3.75 x 5.6 = 21
3.75 x 0.56 = 2.1
Or
0.375 x 56 = 21
0.0375 x 56 = 2.1

**If it is true that:**
**3.75 x 56 = 210**
**What else do you know?**

3.75 x 560 = 2100
3.75 x 5600 = 21000
Or
37.5 x 56 = 2100
375 x 56 = 21000

2.75 x 56 = 210 – 56
1.75 x 56 = 210 – 112
0.75 x 56 = 210 - 168

Can you see how to use this pattern to check the calculation in the middle?

¾ of 56 is 42
¼ of 56 is 14
14 x 4 is 56

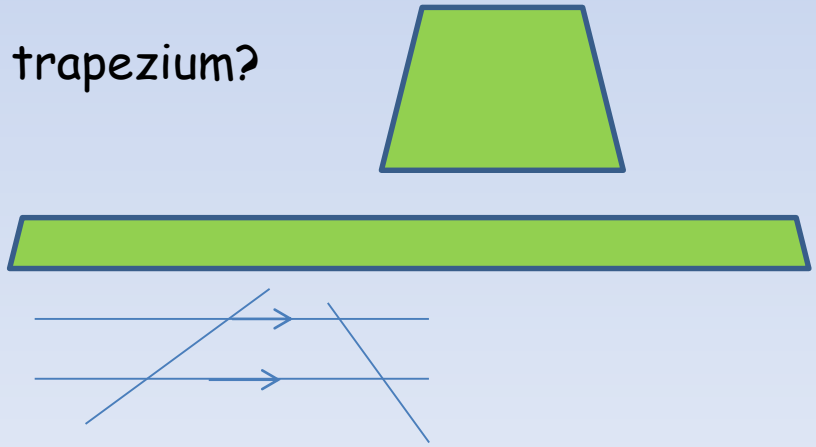
# Particular, Peculiar, General

Pointing to Generality….

Can you give me an example of a fraction that is equivalent to 2/3 ? $\frac{4}{6}$
Can you give me a really peculiar example ? $\frac{2,000,000}{3,000,000}$
Can you give me a general example ? $\frac{2n}{3n}$

Can you give me an example of:
a whole number which leaves a remainder 1 when divided by 3? 4
And a really peculiar example? 3,000,004
And a general example? 3n+ 1

Can you give me an example of a trapezium?
And a really peculiar example?
And a general example?

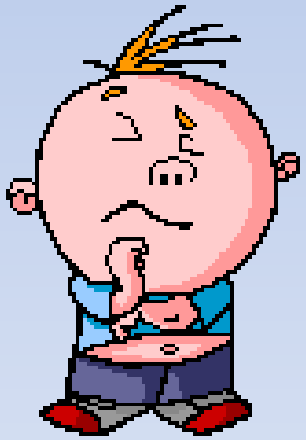


*From 'Thinkers', ATM*

Which questions have I asked / did you identify ?

Have they made you think….

- differently ?
- mathematically ?
- at all ?



What is the same and what is different?

Can you show me an example of ... and another...and another?

# Questions ?

What if I change ……?

If I know ….. then what else do I know ?

Which is harder and which is easier?

# A thought on metacognition

(thinking about thinking, knowing what you are doing and why)

**It is better to have 5 ways to solve 1 problem**
**than 1 way to solve 5 problems.**

Deep understanding of structure enables you to reason and generalise

How many ways can you show me  6 + 15 = 21 ?

Which is your favourite way ?

Which is the ‘best’ way ?

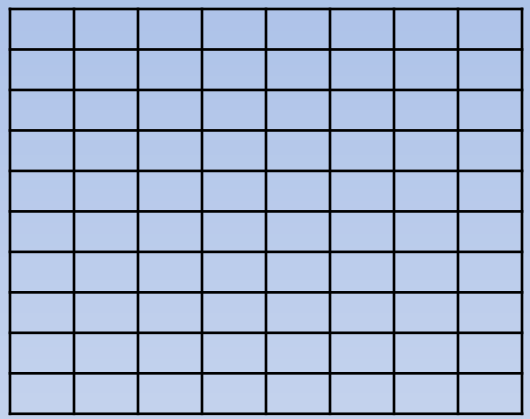
What if I change the 15 to 14? Harder or easier?

10 x 8 = 80

Draw the array

8

10



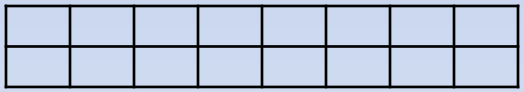
10 x 8 = 80

What if I change the ‘10’ to ‘2’ ?
Draw the array

2 x 8 =

8

2

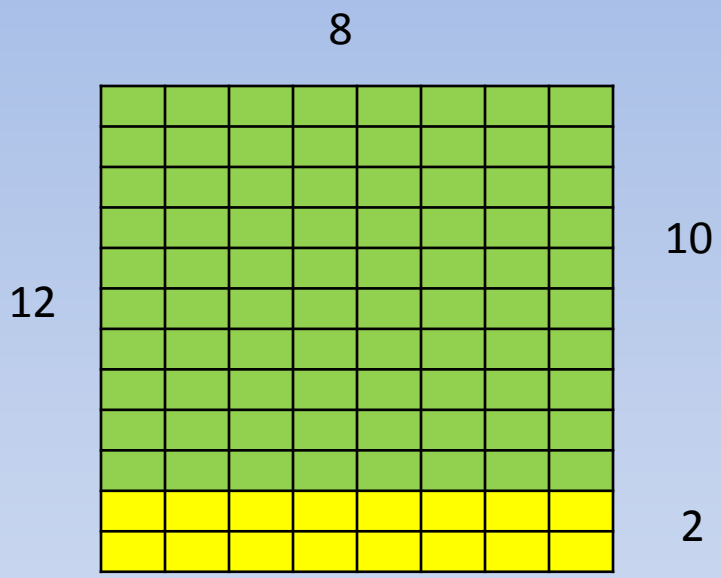


2 x 8 = 16

*From ideas by Mike Askew*

Can you show me 12 x 8 as an array and write it with symbols ?

12 x 8 =




12 x 8 = 96

AND

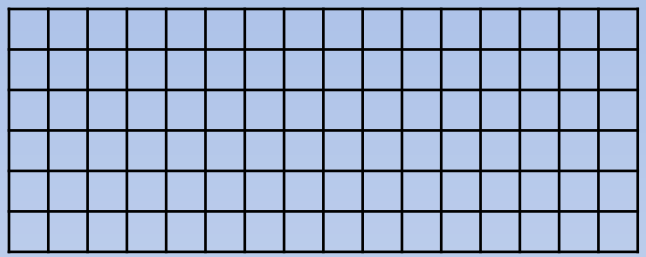
12 x 8 = (10 x 8) + (2 x 8)

*From ideas by Mike Askew*

Can you show me 6 x 16 in a similar way?

16

6

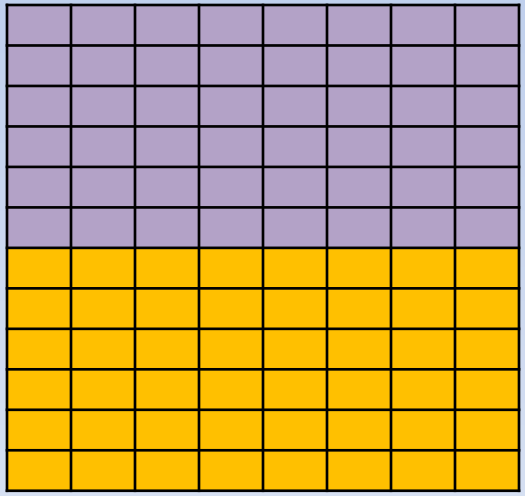


6 x 16 = 96

6 x 16 =

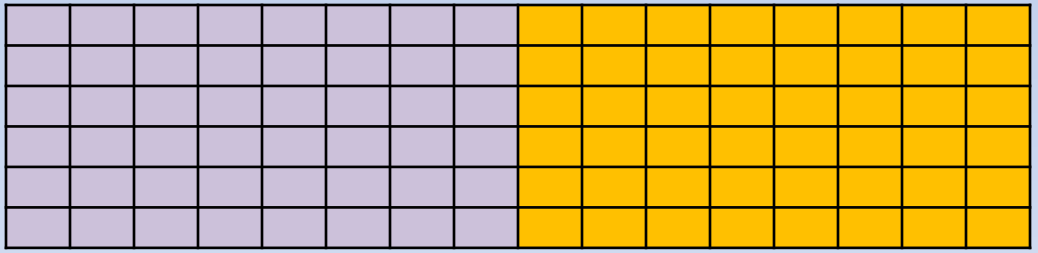
8

12



8 8

6

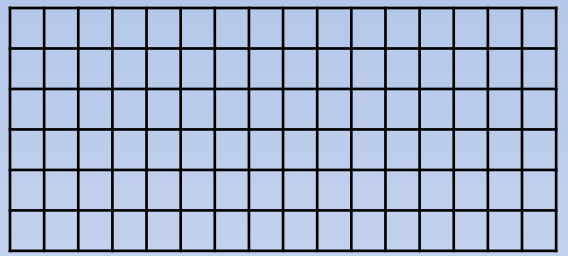


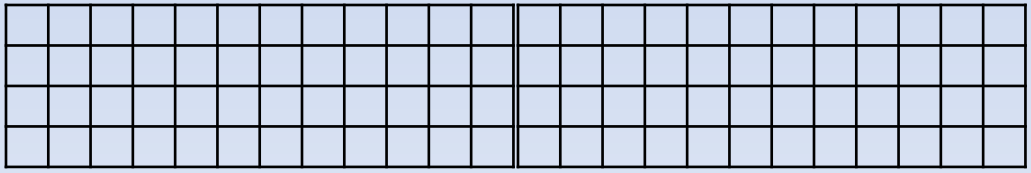
12 x 8 = (10 x 8) + (2 x 8)
AND
12 x 8 = 6 x 16
AND
6 x 16 = (6 x 8) + (6 x 8)

*From ideas by Mike Askew*

Can you show me a different example of a calculation with a product of 96?
And another……?

12 x 8 = (10 x 8) + (2 x 8)
AND
12 x 8 = 6 x 16
AND
6 x 16 = (6 x 8) + (6 x 8)
AND
3 x 32 = (3x8) + (3x8) + (3x8) + (3x8)
AND
4 x 24 = (4 x 8) + (4 x 8) + (4 x 8)



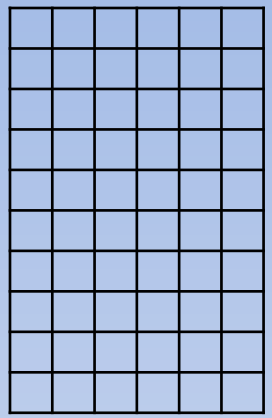


*From ideas by Mike Askew*

If 10 x 6 = 60, then what else do I know ?

Can you show me how to **construct**

- 10 x 6
- 20 x 6
- 30 x 6
- 15 x 12
- 10 x 18
- 5 x 36



How could we record what we have done to show the **structure ?**

- 10 x 6 = 60
- 20 x 6 = 2 x 10 x 6
- 30 x 6 = 3 x 10 x 6
- 15 x 12 = (10 x 6 )+ (5 x 6)
- 10 x 18 = 10 x 6 x 3
- 5 x 36 = 5 x 6 x 6

Other ways ?

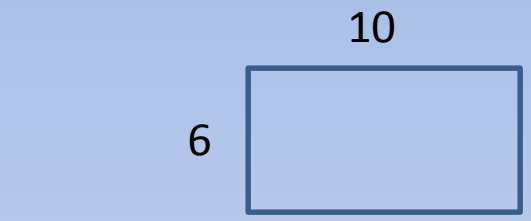
*From ideas by Mike Askew*

## *Using an open array*

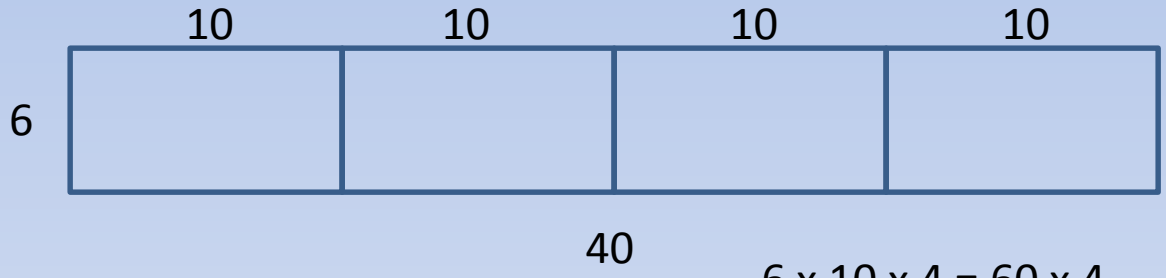
6 x 10

6 x 40

6 x 39




6 x 10 = 60



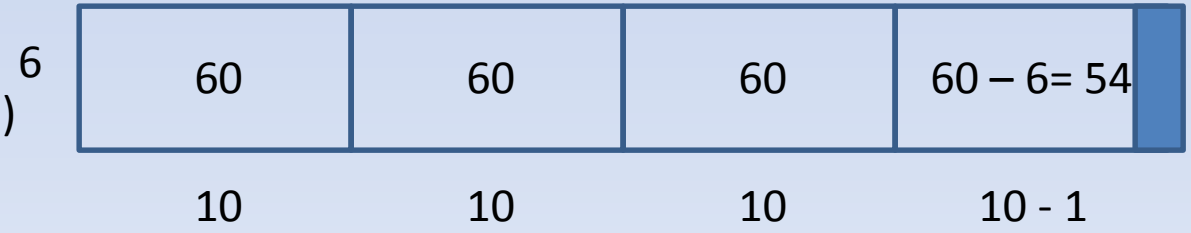

6 x 10 x 4 = 60 x 4

60 x 4 = 6 x 4 x 10

6 x 4 x 10 = 24 x 10

6 x 39 = 6 x (40-1)

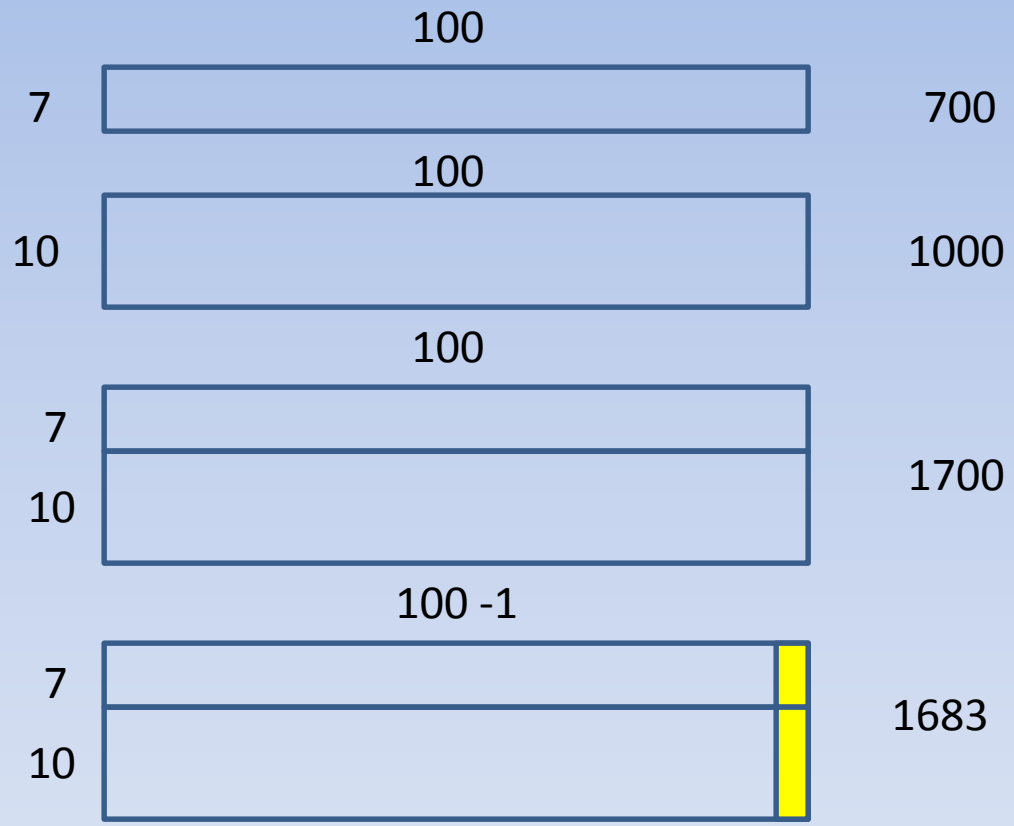
6 x 39 = (6 x 40) – (6x 1)




*From ideas by Mike Askew*

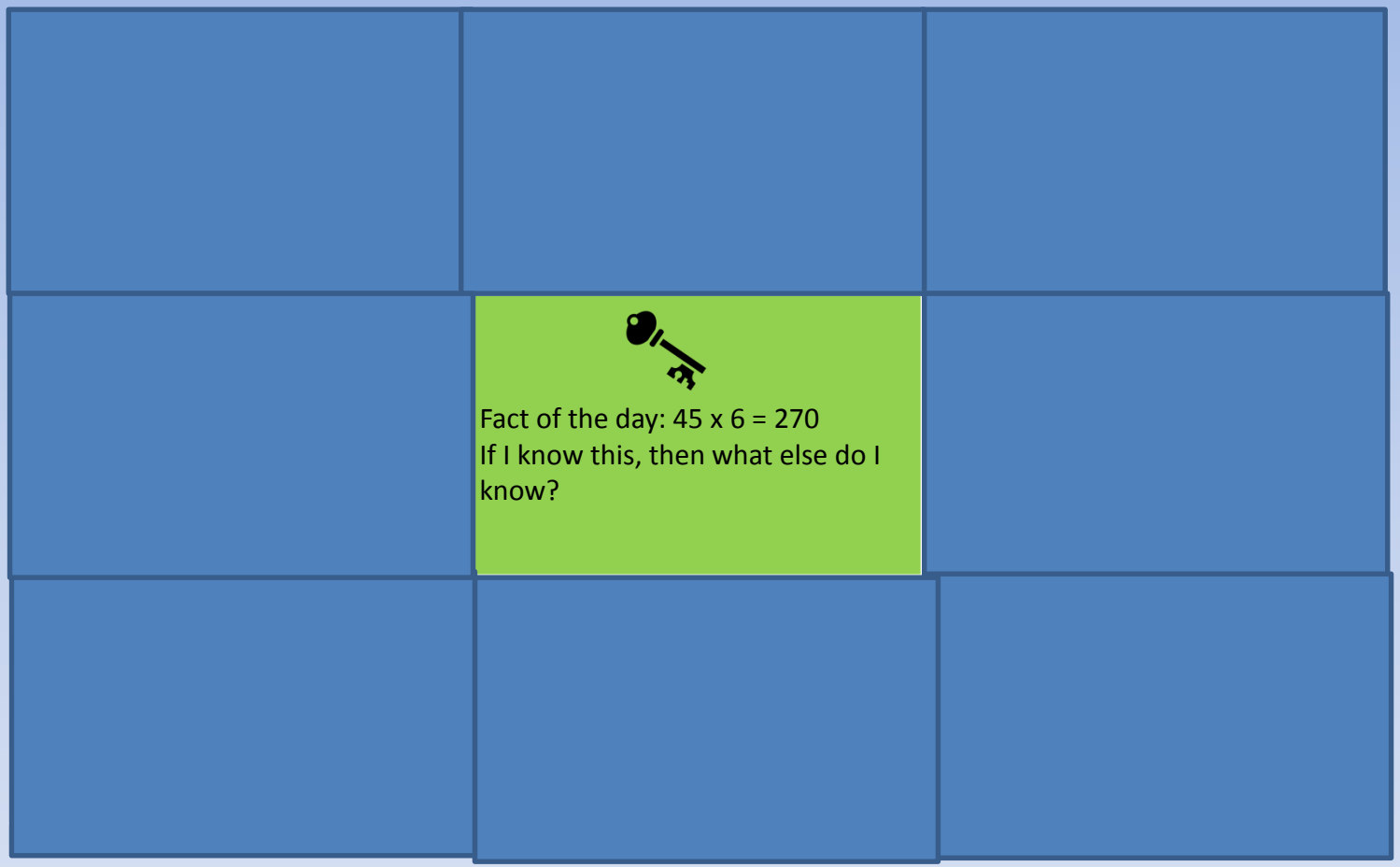
What does the recording look like ?

Another one ?




*From ideas by Mike Askew*

# Fact of the day:
# Linking some ideas/ beginning to put it all together



Fact of the day: 45 x 6 = 270
If I know this, then what else do I know?

## *Number Talks*

# 18 x 5

Solve this with jottings in as many different ways as you can think of.

Have a ‘Number Talk’ with other people and share your ideas and reasoning

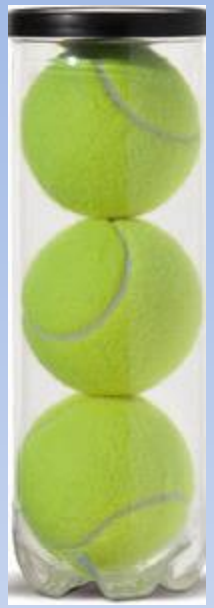
| | | | | |
|---|---|---|---|---|
| **20 x 5= 100**<br>**2 x 5 = 10**<br>**100-10 = 90** | **10 x 5 = 50**<br>**8 x 5 = 40**<br>**50+40=90** | **18x5= 9x10**<br>**9x10 = 90** | **18x2 = 36**<br>**2 x 36 = 72**<br>**18 +72= 90** | **9 x 5 = 45**<br>**45 x 2 = 90** |

Explore the different approaches together to see why they work

*From “Fluency without Fear” by Jo Boaler*

# *Number Talks*

- A sports shop orders 15 boxes of tennis balls.
- Each box contains 8 packs of tennis balls.
- Each pack contains 3 tennis balls.
- How many tennis balls does the sports shop order in total?



Solve this with jottings in as many different ways as you can think of.

Have a ‘Number Talk’ with other people and share your ideas and reasoning

Explore the different approaches together to see why they work

# *Number Talks*

- A sweet shop orders 12 boxes of toffees.
- Each box contains 20 bags of toffees.
- Each bag contains 25 toffees.
- How many toffees does the sweet shop order in total?

Solve this with jottings in as many different ways as you can think of.

Have a ‘Number Talk’ with other people and share your ideas and reasoning

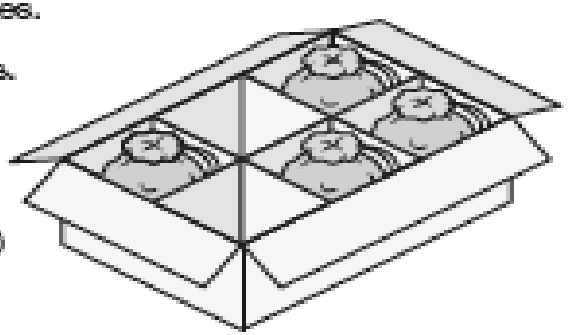
Explore the different approaches together to see why they work

# *Number Talks*

**11** A toy shop orders 11 boxes of marbles.

Each box contains 6 bags of marbles.

Each bag contains 45 marbles.

How many marbles does the shop order in total?

Show your method

marbles

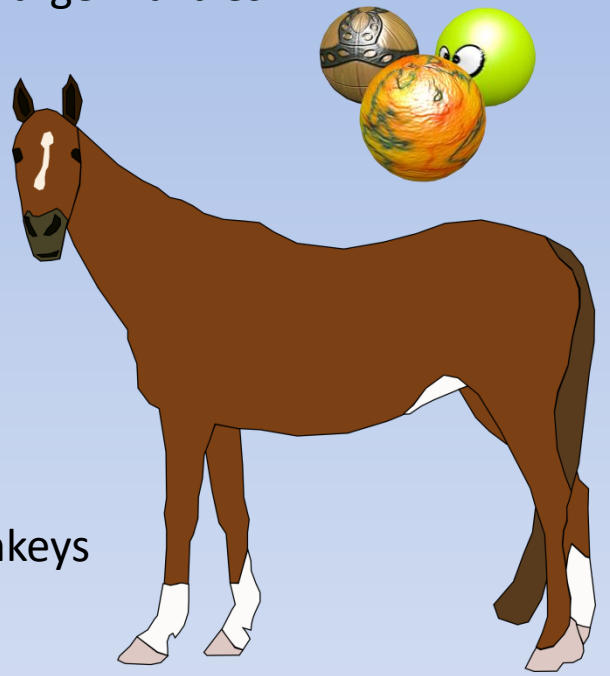
2 marks

# *Number Talks*

20 small marbles have the same mass as 5 large marbles

The mass of one small marble is 1.5 g

What is the mass of one large marble ?

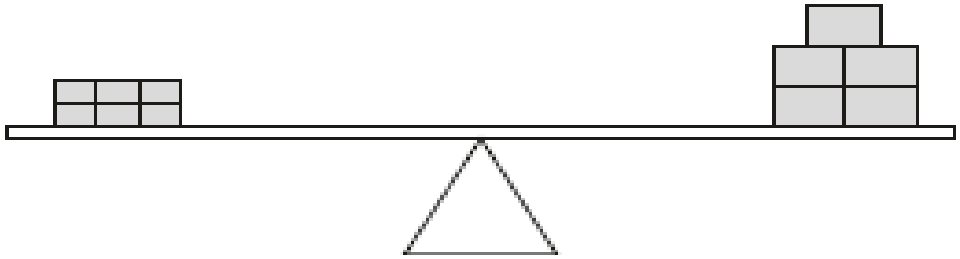


18 horses have the same mass as 33 donkeys

The mass of one horse is 550kg

What is the mass of one donkey?

**16** 6 small bricks have the same mass as 5 large bricks.



The mass of one small brick is 2.5 kg.

What is the mass of one large brick?

Show your method

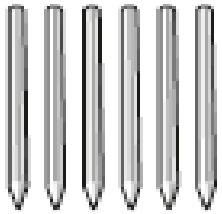
kg

2 marks

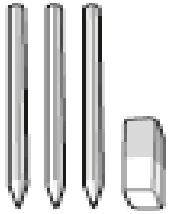
What sequence of learning is needed to support pupils to be successful with this one?

**9**

6 pencils cost **£1.68**



3 pencils and 1 rubber cost **£1.09**



What is the cost of **1 rubber**?

Show your method

2 marks

Final thoughts : what are our conclusions ?

Learners need to feel positive when they are problem solving and reasoning.
They need to be sure that the solution is out there somewhere!

How do we do this?

Developing an understanding of structure through appropriate models , images
and multi-representations

Building fluency and familiarity with types of problem rather than testing what is not known

Construct a sequence of learning to support access and success for all

Giving learners time to work individually and collaboratively to make sense of the mathematics

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