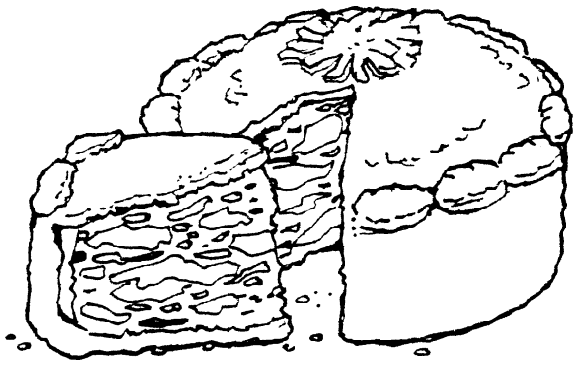


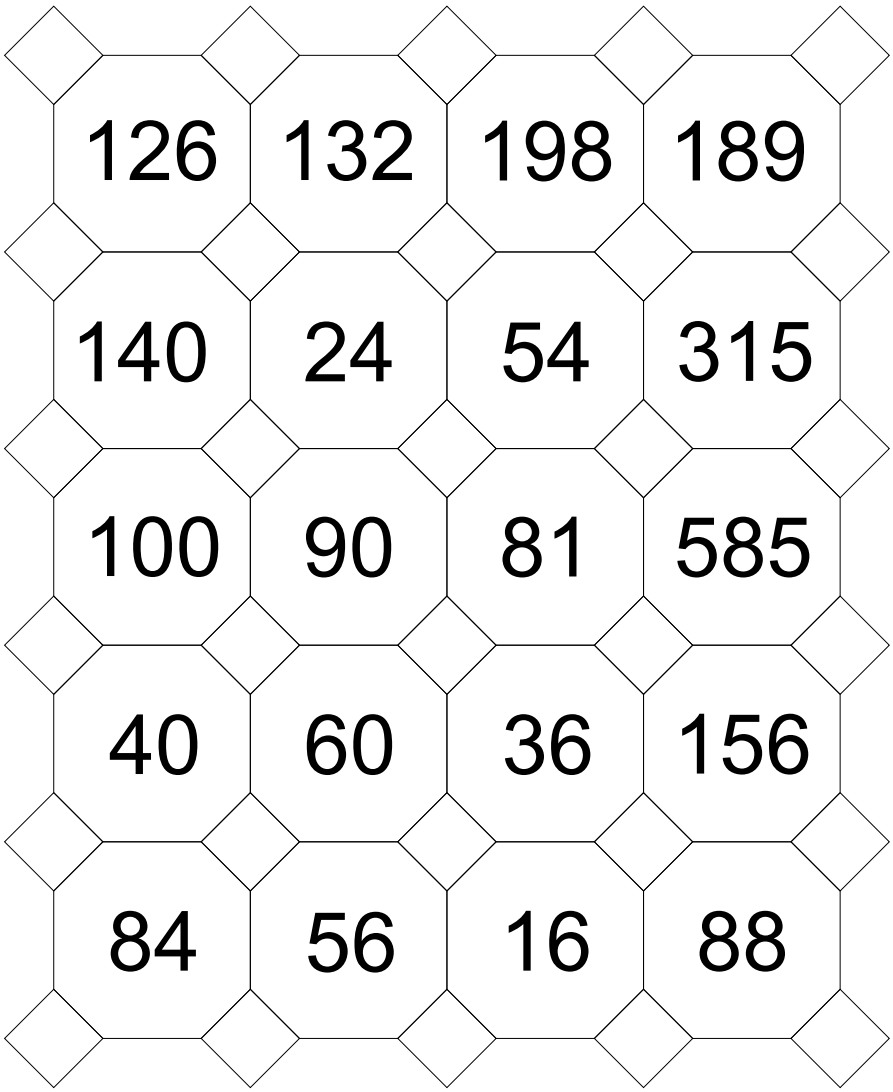
MATHEMATICAL PIE

No 178 The Mathematical Association 259 London Road, Leicester LE2 3BE **AUTUMN 2009**

Prime Factor Tessellation

Each of these numbers can be expressed as a product of four prime numbers. Work out the prime factors and then arrange them so that each number is surrounded by its prime factors

V.L.



Alphapower (Go West?)

Each letter stands for one of the digits 0 to 9. For example, if B is 8 and G is 5, GB would mean 58 and G^B would be 5^8.

If D^D = ROAD and H^D = RANCH, find D, R, O, A, N, C and H.

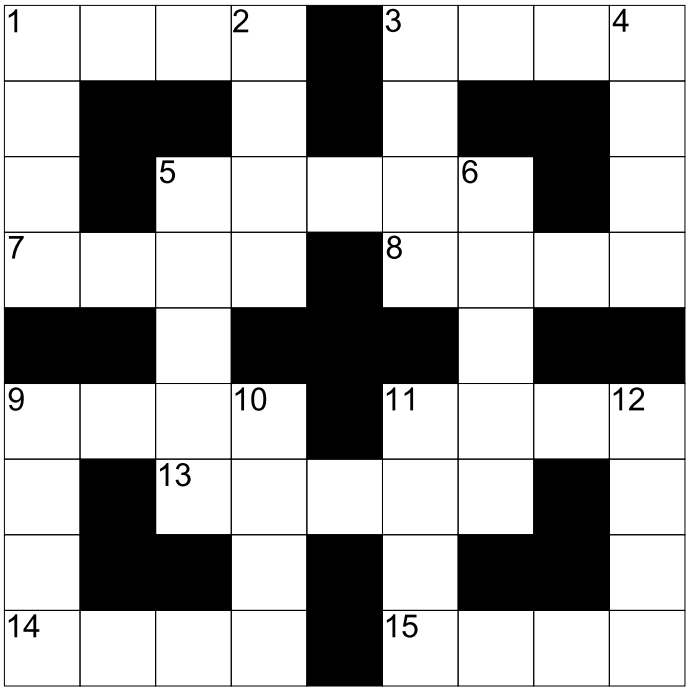
If I also tell you that D^E = CAD, does this make the puzzle N^N?

E.G.

Alphabetical Crossword

In this crossword the answers are words. A=1, B=2 etc and these numbers are placed next to each other to make one long number.

W.R.

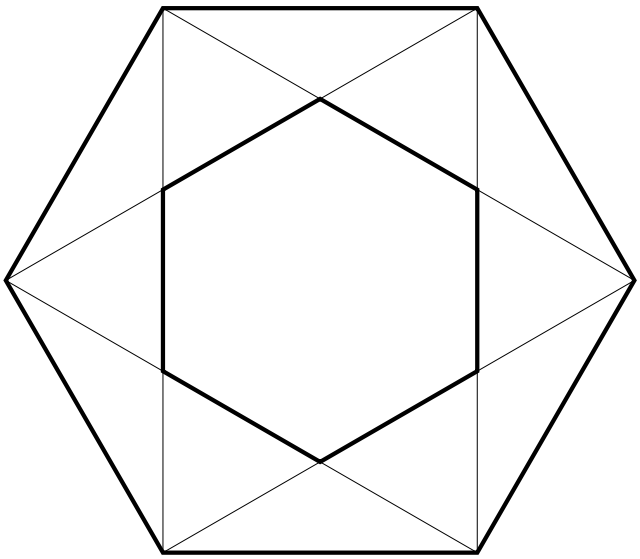


Across		Down	
1.	16211420	1.	1615144
3.	1911220	2.	20954
5.	19522514	3.	1920516
7.	4554	4.	201165
8.	1615125	5.	1959265
9.	131265	6.	14152125
11.	2121523	9.	131912
13.	517125	10.	511920
14.	125620	11.	212215
15.	522912	12.	2311212

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Regular Hexagons Again

Find the simple relationship between the areas of the inner and outer hexagons:



H.K.M.

Prime or Not Prime?

1234567 − 127 =1234440.

Find the prime factors of 123444.

Now you should be able to answer the question:

Is 1234567 prime?

B.G.S.

A Square Octet

1	7	4	12
5		2	
3	6	8	17
9		14	

The numbers 1 to 8 have been placed at random in the diagram. The totals of the two rows and the two columns are shown.

It is possible to rearrange the numbers 1 to 8 so that these four totals become the same: I have found six different solutions. Can you do better?

(Of course I do not count rotations or reflections as different!)

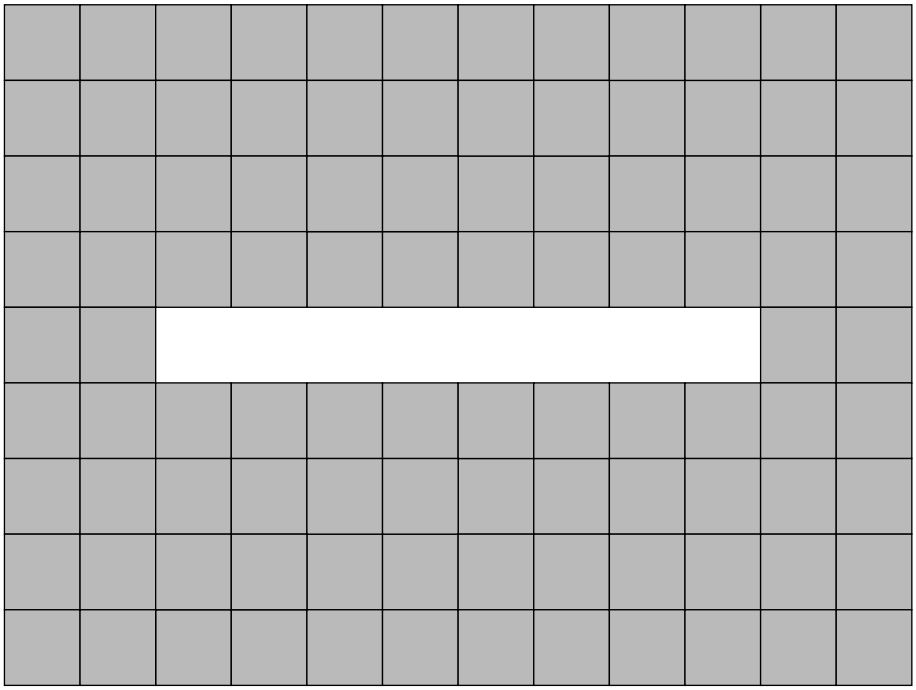
E.G.

Dudeney's Damaged Patchwork Quilt

The Englishman H.E. Dudeney (1857-1930) is famous for many great puzzles. Here is one of them.

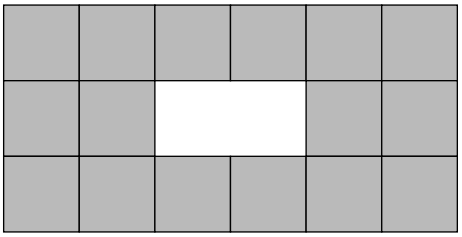
A patchwork quilt consists of 9 squares by 12 and is damaged by having eight of its squares missing:

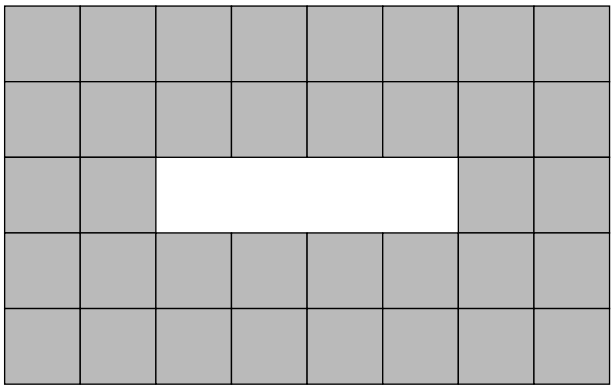
How can the damaged quilt be repaired by making two cuts and rearranging the two parts into a 10 by 10 square?

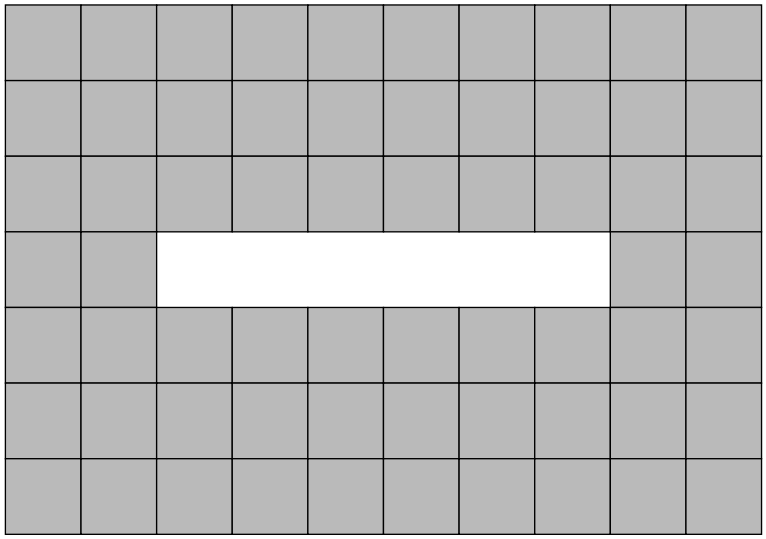


Dudeney's puzzle can be generalised to any odd-by-even quilt, where the even side is three squares longer than the odd side, with some squares missing from the centre and the solutions are basically the same. In the next issue, we will look at damaged quilts which are even-by-odd.

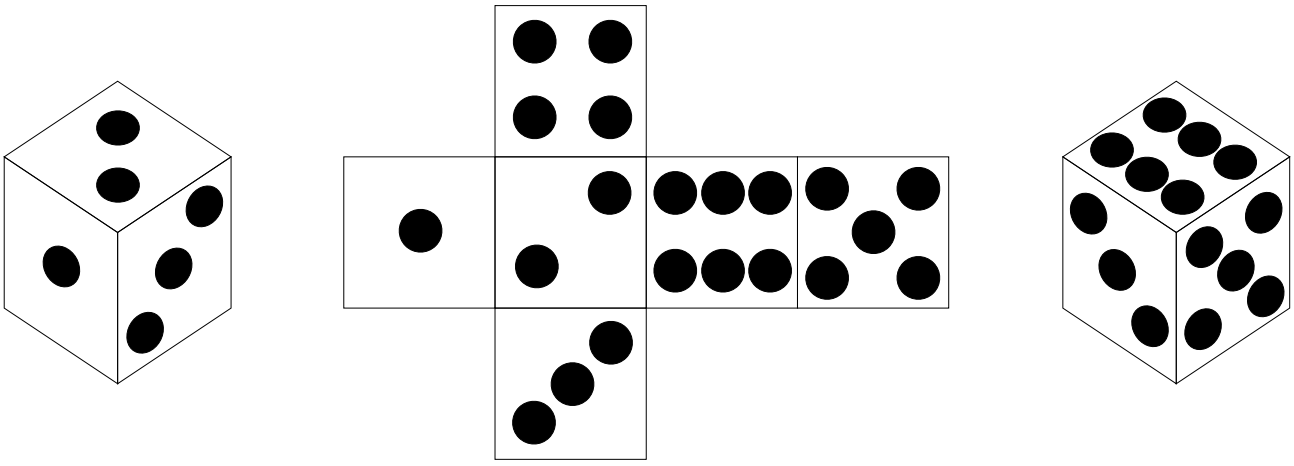
W.R.







The 8-Dice Cube



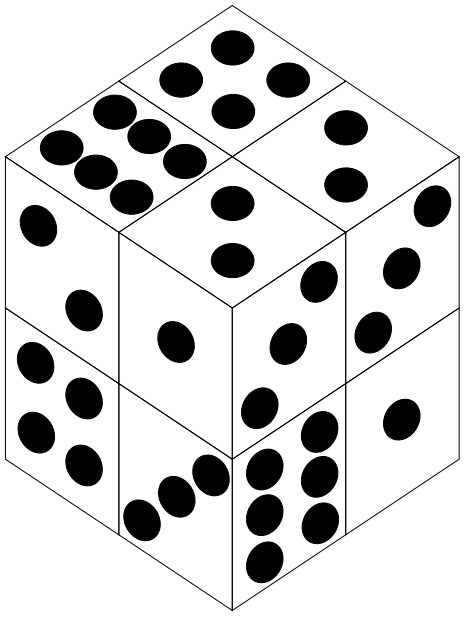
Here are two views and the net for a ‘normal’, ‘right’ or ‘clockwise’ die – where one view of the die shows the faces 1, 2 and 3 running in a clockwise fashion.

Suppose that you have 8 identical clockwise dice. If you can find 8, good – if not then maybe you will need to do as I did and make a set from thin card.

The 8 dice can be arranged into a “2-by-2” cube. Obviously each die will then have three faces showing and three faces hidden within the large cube. The cube shown below has 14 spots on the top face, 13 on the right and 10 on the left (there is no knowing from the diagram what the totals on the other faces are, but with a real 8-dice cube you would be able to look at them all).

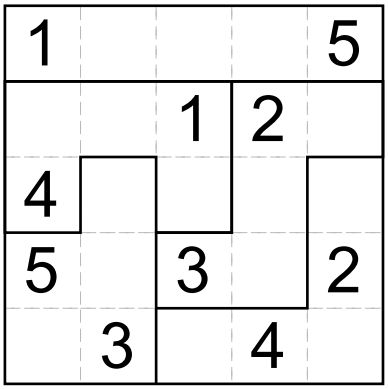
Can you find ways of arranging the 8 dice so that the total on each of the six faces of the large cube is the same? It is not very hard to make the totals on each face 8, 12, 16 or 20. How many of the other potential totals between 8 and 20 can you get? Can you get any totals below 8 or above 20?

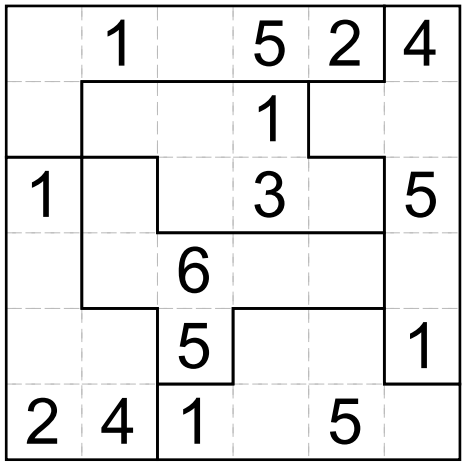
E.G.



Pentomino and Hexomino Sudoku

Each number from 1 to 5 must appear in each row, column and pentomino on the left, and similarly for the hexomino puzzle.





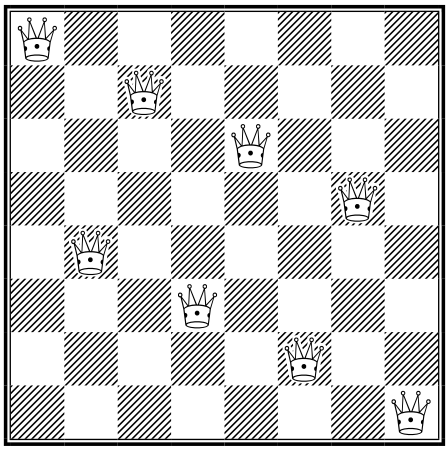
V.L.

Queens and Bishops

It is well-known that 8 queens are the most that can be placed on an empty chessboard so that none is attacked by any other, though it's not so simple to do.

Can you?

Here is a near miss:



Bishops are easier, and more are possible. How many?

H.K.M.

Thick?

When I was reading my Sunday newspaper I found that my wife had taken one complete page to do the crossword. The missing pages were 17, 18, 63 and 64.

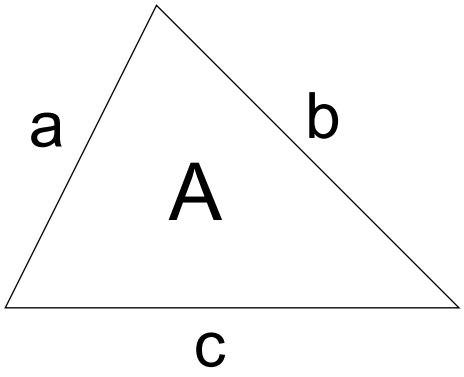
How many pages did the complete paper have?

E.G.

Heron's Formula

Half-perimeter "s" soon provides
Triangle facts, Heron decides
"Area squared, I would guess--
Is the product of s
And its difference with each of the sides."

Heron was one of the mathematicians working in Alexandria in the first century A.D. He worked on both numerical and geometrical ideas. For example, he suggested $7 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$ as an approximation to the square root of 63. If you try it you will see that he was very close! The formula that bears his name – for the area of a triangle – was probably discovered earlier by Archimedes though Heron developed it from a geometrical point of view. It has a pleasing symmetry:



$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{1}{2}(a+b+c)$$

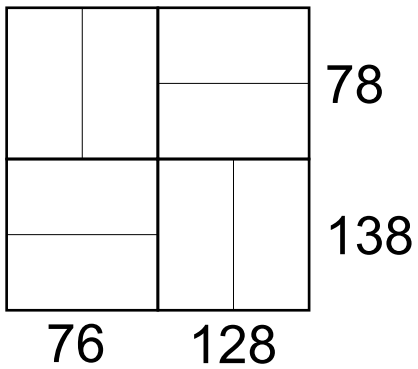
E.G.

Yes but which?

Fill the half-squares with numbers between 1 and 9.

Reading across and down, you must choose three numbers which, when written in the form A × BC, give the total on the left or at the bottom (thus 436 will give 4 × 36 and 144 will be written at the side).

For across answers, if a square is split vertically use both numbers; if horizontally choose just one of them. For down answers, if a square is split horizontally use both numbers; if vertically choose just one of them.



W.R.

Long Before Sudoku

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

The American puzzle inventor Sam Loyd has been described as a genius. If you look at a book of his inventions you may agree. In 1878 he invented "The 15 Puzzle", a square frame in which numbered tiles may be moved around via a single vacant place. In the diagram only 12 or 14 can move to begin with.

The game was used as a marketing device for various products, with prizes for obtaining various configurations. Some employers forbade their workers from playing the game during working hours, but Sam Loyd made quite a lot of money.

There are more than 2×10^{13} theoretical arrangements, but it has been proved that only about half these can be reached from the starting position shown; a prize of $1000 was offered to anyone who could put the numbers in their natural order (in other words simply reverse 14 and 15 in the diagram above), but it was safe.

Here's a much easier question: How many moves are necessary from this starting position to get the 1 into the central square? (No drawing allowed!)

H.K.M.

1	2	3
4	5	6
7	8	

MATHEMATICAL PIE (three issues per year)

Single copy: yearly subscription £4.00
Multiple copies (packs of ten):1–9 packs: £12.00 per pack
10–19 packs: £10.00 per pack
20 or more packs: £8.00 per pack

The Mathematical Association,
259 London Road, Leicester LE2 3BE
Tel: 0116 221 0013
Fax: 0116 212 2835
E-mail: office@m-a.org.uk; website: www.m-a.org.uk

MATHEMATICAL ASSOCIATION
supporting mathematics in education