Primary Mathematics Challenge – February 2014

Answers and Notes

These notes provide a brief look at how the problems can be solved.
There are sometimes many ways of approaching problems, and not all can be given here.
Suggestions for further work based on some of these problems are also provided.

P1 **C** 3 P2 **C** $(2 + 0) \times 14 = 28$

1 **C** 5 — Every number whose units digit is 5 is a multiple of 5, so no number greater than 10 can be prime. On the other hand, 11, 23, 37 and 59 are examples of prime numbers ending in 1, 3, 7 and 9.

2 **B** 21 — Of the factor pairs for 90 (1 × 90, 2 × 45, 3 × 30, 5 × 18, 6 × 15 and 9 × 10) only 6 and 15 have a difference of 9; their sum is 21.

3 **C** 15 cm^3 — The original cube has a volume of $3 \times 3 \times 3 = 27\ \text{cm}^3$, from which are removed four cuboids each of volume $3\ \text{cm}^3$. So the remaining solid has volume of $27 - 3 \times 4 = 15\ \text{cm}^3$. Alternatively, the cross-sectional cross has an area of $5\ \text{cm}^2$ and length 3 cm, hence the volume is $5 \times 3\ \text{cm}^3$.

4 **E** 24 — If the car can travel 48 miles with 1 gallon of petrol, half a gallon will take it 24 miles.

5 **A** 12 — With three teachers missing, 20 are at the staff meeting. Five are asleep and, of the remaining 15, three (20%) are texting. This leaves 23 – 3 – 5 – 3 = 12 teachers listening to the Head.

6 **C** 35° — Because triangle *RQS* is isosceles, angle *RSQ* = angle *SRQ* = 70°. Therefore we know that angle QSP = 180 – 70 = 110°. Triangle *QSP* is also isosceles, and so angle *SQP* = x°. Now, considering the three angles of triangle *QSP*, we have $x + x + 110 = 180$, and so $x = 35°$.

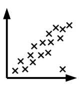
7 **D** 400 ml — Alice has 10 litres = 10 000 ml, which divided by 25 is 400 ml.

8 **B** 2 — Because Dhiran's code is a multiple of 3, its digits must have a total that is a multiple of 3. The sum of 4, 5 and 1 is already 10, so the fourth digit could be 2, 5 or 8. Since 4515 is a multiple of 5 and 4518 is a multiple of 9, Dhiran's code is 4512.

9 **B** one quarter — One way to tackle this question is to start with a number of strawberries for which we can easily find one third, one quarter and one sixth: 12 is the smallest. Then 4 are eaten by wasps, 3 by ants and 2 by a maggot, leaving 3 out of 12, or one quarter. Alternatively, one can think in fractions, though this amounts to the same thing:

$$1 - \frac{1}{3} - \frac{1}{4} - \frac{1}{6} = \frac{12}{12} - \frac{4}{12} - \frac{3}{12} - \frac{2}{12} = \frac{3}{12} = \frac{1}{4}$$

10 **E**



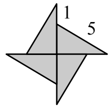
According to Polly's observation, her graph should show children who do more sports to be good at football, and those who do less to be not so good, and with one exception. Graphs A and D show no connection (or *correlation* as mathematicians refer to it); graph B shows the very opposite of Polly's conclusion, and of graphs C and E, only E has an exception.

11 **E** right-angled and scalene — The three angles of a triangle add up to 180°. In order for the ratio of the angles to be 1 : 2 : 3, the largest has to $\frac{3}{6} = \frac{1}{2}$ of the total, and therefore 90°. Since the smallest two angles are not equal, the triangle must be right-angled and scalene (its angles being 30°, 60° and 90°).

12 **B** 101 000 — Changing both masses into grams, the number of times Clara is heavier is $2020 \div 0.02$, which might be thought of as $2020 \div \frac{1}{50} = 2020 \times 50 = 101\,000$.

13 **C** 550 g — We can tell that one-third of the plum jam weighs $400 - 250 = 150$ g (without the pot itself). Hence the pot on its own weighs $250 - 150 = 100$ g, and a full pot of jam weighs $150 + 3 \times 150 = 550$ g.

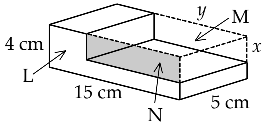
14 **C** 24 cm — The longest side of each triangle is 5 cm and each of the shorter straight parts of the perimeter are $4 - 3 = 1$ cm long. Therefore the perimeter of the whole shape is $4 \times (5 + 1) = 24$ cm.



15 **D** 24 weeks — There are 5 choices for who goes first in line, and after that 4 choices for who goes second, and then 3 for third, 2 for fourth and then 1 choice for who will have to be last: so $5 \times 4 \times 3 \times 2 \times 1 = 120$ ways. This means that they not have to repeat until after 120 days, or 24 school weeks.

16 **D** 12.18 am — The time passed between 6 pm and 1 am is 7 hours, and in this time the second clock will have lost $7 \times 6 = 42$ minutes. So the time it will show is 12.18 am.

17 **C** 3 and 12 — If one imagines the shape (L) as a cuboid with a smaller cuboid (M) cut from it as shown on the right, the volume of the larger cuboid is $4 \times 5 \times 15 = 300$ cm^3. However, what is left (L) has a volume of 120 cm^3, so that shape M has a volume of $300 - 120 = 180$ cm^3. This means its cross-section (shaded N) has an area of $180 \div 5 = 36$ cm^2. All of the pairs of dimensions will give an area of 36 cm^2, but only the pair $x = 3$ and $y = 12$ are smaller than the upright 4 cm and length 15 cm.



18 **A** 94 — The largest single-digit prime number is 7 and the smallest three-digit prime number is 101; their difference is $101 - 7 = 94$.

19 **D** 56 cm^2 — The area of the octagon is the area of the square less the area of 4 corner triangles, so $8 \times 8 - 4 \times 2 = 56$ cm^2.

20 **D** 6 — Because 2, 19 and 53 are prime factors of 2014, the only other factors are these factors themselves or products of some of them. So there are 6 factors apart from 1 and 2014: 2 and 1007, 19 and 106, 38 and 53.

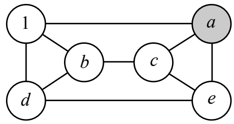
21 **A** 3 hours 30 minutes — If Anita can tidy 1 big and 3 small rooms in 90 minutes, she could manage 2 big and 6 small in $90 \times 2 = 180$ minutes. But 2 big rooms take her as long as 3 small rooms, so that she could tidy $3 + 6 = 9$ small rooms in 180 minutes. Thus one small room will take her $180 \div 9 = 20$ minutes, and so 3 big rooms and 6 small rooms will take her $3 \times 30 + 6 \times 20 = 210$ minutes = 3 hours and 30 minutes.

22 **C** 4 — Knowing that $p = 3$, $q = 2$ and $p^3 \times q^2 \times r = 432$, we can substitute the letters p and q for the numbers they represent. So $3^3 \times 2^2 \times r = 27 \times 4 \times r = 432$ and hence $r = 432 \div (27 \times 4) = 432 \div 108 = 4$.

23 **B** 96 — Going from left to right from one house to the next, the house numbers on the top row increase by one, but the house numbers on the bottom row decrease by one each time. This means that the total of the numbers of houses opposite each other remains the same for each pair of facing houses. For the pair of houses we know about this total is $46 + 145 = 191$, but we also know that the shaded house and the house facing it have consecutive numbers. Therefore they must be numbered $(191 - 1) \div 2$ and $(191 + 1) \div 2$, that is, 95 and 96.

24 **D** 56 — There are only four positive cube numbers less than 100: 1, 8, 27 and 64. Lily is 64, Jilly is 27, Milly (the elder sister) is 8 and Tilly is 1. In this way, when Milly was born, Lily would have been $64 - 8 = 56$ years old.

25 **B** 4 Since we know where the 1 is placed, the possibilities for placing the number 2 are rather limited, so one approach is to explore where the 2 can be placed. We shall refer to the circles as in the diagram below:



It is clear that 2 must be placed in either c or e, since a, b and d are joined to 1.
If 2 is placed in c, then $d = 3$, $a = 4$, $b = 5$ and $e = 6$.
Alternatively, if 2 is placed in e, then $b = 3$, $a = 4$, $e = 5$ and $d = 6$.
In both cases, the number that is placed in circle a is 4.

Some notes and possibilities for further problems

Q1 When we think about the *units* digits of prime numbers, we are also considering what remainder there could be when dividing by ten. You might also consider what remainder there might be if instead you divide a prime number by 6: for primes greater than 3 the remainder cannot be an even number (as then the prime would also be even); nor can it be 3 (as then the prime would be a multiple of 3). Hence the only remainders you can get when dividing a prime number by 6 are 1 or 5 (mathematicians say that prime numbers greater than 3 are either $6n + 1$ or $6n + 5$).

Q7 The author of *Alice in Wonderland* (where Alice meets the Hatter at the Mad Tea-Party) is Lewis Carroll, the pen-name of the mathematician Charles Lutwidge Dodgson (1832-98) who taught at Oxford for 26 years and worked in the areas of geometry and logic. The *Carroll diagram* is named after him: an example is shown above on the right.

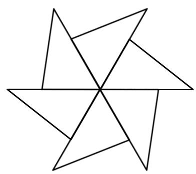
	factor of 12	not a factor of 12
prime	2 3	5 7 11
not prime	1 4 6 12	8 9 10

categorising the positive integers up to 12

Q11 What type of *quadrilateral* has angles in the ratio of 1 : 1 : 1 : 1, or 1 : 2 : 1 : 2, or 1 : 2 : 2 : 1, or 1 : 4 : 3 : 4, or 1 : 7 : 1 : 3 ? What about the *pentagon* whose angles are in the ratio 3 : 5 : 2 : 5 : 3 ?

Q12 It is fun finding comparisons like the one here, and often the answers are surprising. You could compare the height of one of Britain's shortest mountains, Snaefell on the Isle of Man (621 m high), with that of Mount Everest (8848 m high). Or, maybe, compare the population density of London (12 000 people per sq. mile) with that of Greenland (15 sq. miles per person). How about the mass of an acorn (about 4 g) and the mass of a large oak tree (about 8 tonnes)? And don't forget the 220 km length of Britain's longest canal, the Grand Union Canal, set against the shortest, the Wardle Canal in Cheshire, which is a mere 47 m long!

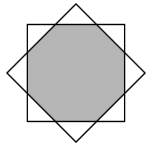
Q14 The triangle with sides of 5 cm, 7 cm and 8 cm has a 60° angle between the 5 cm and 8 cm sides, and so, with that angle at the centre, it is possible to fit together 6 of them together to form a shape with rotational symmetry, as shown in the diagram on the right. What is the length of its perimeter?

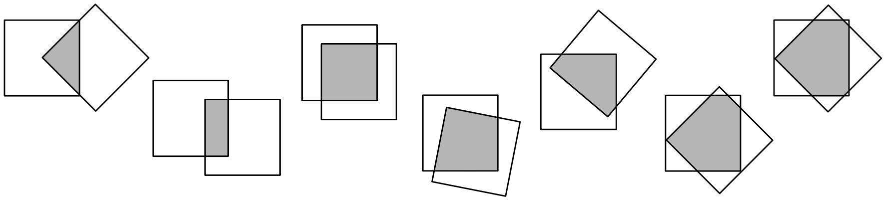


Q15 This question involves a part of mathematics called *permutations*. Here 5 girls can be arranged in any order, and the number of different ways is 5 × 4 × 3 × 2 × 1 = 120. This idea comes up so often that there is a short way of writing it using an *exclamation mark*! In this way 5! = 120 (pronounced "5 factorial") – you may have a button on your calculator that will calculate these for you, though the numbers produced become very large very quickly. How many different ways are there of arranging all the letters of the word TRIANGLE into another "word", even one that is not a word in any useful language?

Q18 On the other hand: what is the difference between the largest single-digit **non**-prime number and the smallest three-digit **non**-prime number?

Q19 Using the same two 8 cm squares, the smallest area possible for the octagonal overlap is when the octagon is regular: the octagon then has an area of just over 53 cm^2. But another interesting question to ask is what other polygons can be made from the overlap of two identical squares? Some possibilities are shown below:





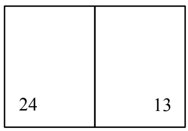
Can you find different polygons with the overlap if you allow the squares to be different sizes?

Q20 Not only is 2014 = 2 × 19 × 53, but last year 2013 = 3 × 11 × 61 and next year 2015 = 5 × 13 × 31; they each have 8 factors. When is the next year number to have exactly 8 factors? You'll be glad to know that you have not got to wait until 4102 = 2 × 7 × 293.

Q23 The idea of totals remaining the same is a very useful notion – it occurs in the famous story of the mathematician Gauss (1777-1855), who in school was asked by his frustrated teacher to work out

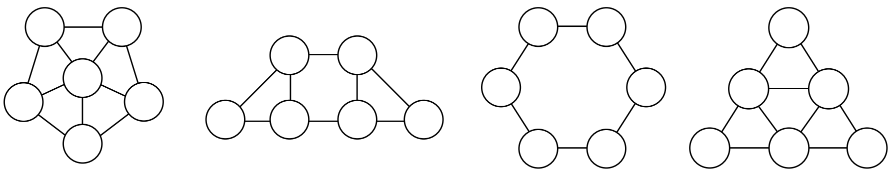
$$1 + 2 + 3 + 4 + \ldots + 97 + 98 + 99 + 100.$$

Gauss did so by noticing that 1 + 100 = 101, 2 + 99 = 101, 3 + 98 = 101, and so on, hence making the calculation equivalent to the sum of 50 pairs of numbers with a sum of 101, that is 50 × 101 = 5050. It also has interesting consequences in the numbering of the pages of newspapers, where a single folded sheet of paper has two printed pages on each side. It can be seen that, like this, the two page numbers on each side will have the same total for every side of every sheet. On one side of one of the inside sheets of my newspaper today the page numbers are as shown below:



Can you tell from this how many pages there are altogether, and how many sheets of paper are needed for the whole newspaper?

Q25 The answer to the question gives two possible ways to arrange the numbers 1 to 6 in a *network* so that the difference between any two numbers is always greater than 1. Can you do the same for the networks shown below – if you can, show how it can be done; if you cannot, explain why it is impossible (you can choose where to put the number 1 in any of them)?



The PMC is organised by The Mathematical Association

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