Using A level mathematics to model infection spread and control

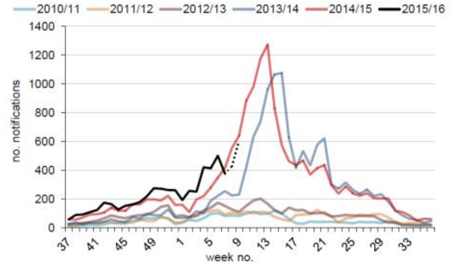
K A Jane White
Department of Mathematical Sciences
University of Bath

LONDON MATHEMATICAL SOCIETY EST. 1865

The Context

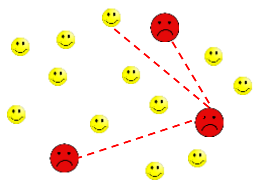
The data: Scarlet Fever

Figure 1. Weekly scarlet fever notifications in England, 2010/11 onwards*



* Dashed line indicates that numbers may increase as further notifications expected.

Infection Process



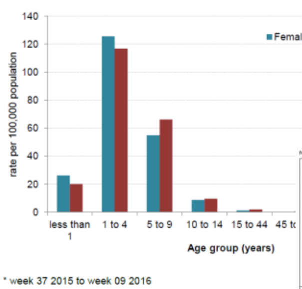
Infection	R0
Measles	12 - 18
Ebola	1.5 - 2.5
Chicken pox	10 - 12
Scarlet fever	5 - 8
HIV	2 - 12

Average number infections from single infected in a population

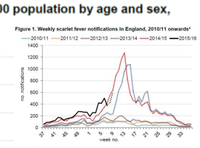
R_0 = Average number contacts per unit time
x Probability contact results in infection
x Time infected

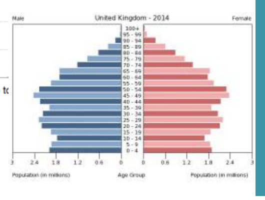
Data in context

Figure 2. Rate of scarlet fever notifications per 100,000 population by age and sex, England, 2015/16*



* week 37 2015 to week 09 2016





Infectious disease dynamics

Population of size N = constant

Susceptible → *Infected* → *Removed*

N = S+I+R

Book-keeping principle

Number of infected individuals in region R at time t
= Number of infected individuals in region R at time t_0
+ Number of new infections in region R in interval $[t_0, t]$
- Number of infected individuals that recover in interval $[t_0, t]$

Model equations

Number of new infections per week

= R_0 x Number infected x Number susceptible to infection

$$u = \frac{S}{N}\,; \quad v = \frac{I}{N}$$

$$\frac{du}{dt} = -R_0 uv \qquad \frac{dv}{dt} = R_0 uv - v = (R_0 u - 1)v, \qquad u + v \le 1$$

Start of infection outbreak

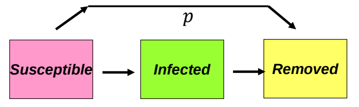
$$u \approx 1 \qquad \frac{dv}{dt} \approx (R_0 - 1)v$$

$$\boldsymbol{v(t) \approx v_0 e^{(R_0 - 1)t}}$$

$R_0 > 1$ infection outbreak

$R_0 > 1$ infection dies out

Public health intervention



p = Fraction removed from susceptibles per unit time

$$\frac{du}{dt} = -R_0(1-p)uv \qquad \frac{dv}{dt} = R_0(1-p)uv - v = (R_0(1-p)u - 1)v$$

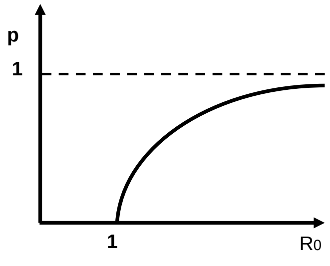
Herd immunity when R_0>1

Value of p for which an initial outbreak is contained

$$u \approx 1 \qquad R_0(1-p) - 1 < 0 \qquad \boldsymbol{p > 1 - \frac{1}{R_0}}$$

Fixed intervention strategy

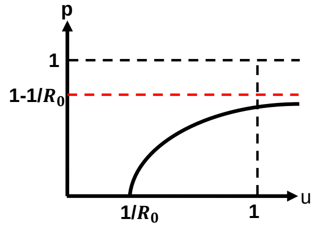
$$\boldsymbol{p > 1 - \frac{1}{R_0}}, \qquad \boldsymbol{R_0 > 1}$$



Improved strategy

Allow p to vary as infection passes through population

$$R_0(1-p)u - 1 < 0$$



Control strategy

Minimise cost of infection

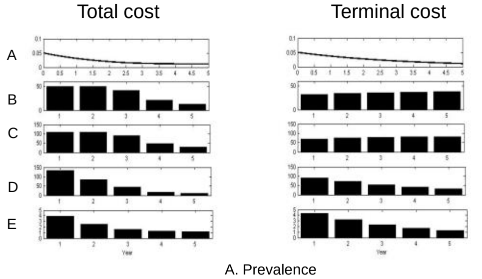
$$J(\boldsymbol{p}) = \int_0^T C_1\, N|v(t) - v_c| + C_2 N p(t) R_0 v(t)\, dt$$

C_1 = Per capita Cost of infection (treatment, time off work)
C_2 = Per capita Cost of intervention

subject to

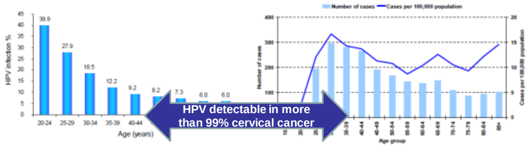
$$\frac{du}{dt} = -R_0(1-p)uv \qquad \frac{dv}{dt} = R_0(1-p)uv - v$$

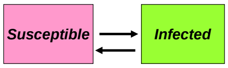
Impact of cost choice



A. Prevalence
B. Percentage population screened
C. Screening cost
D. Infecteds identified via screening
E. Percentage screens identifying infecteds

HPV and vaccination





Outcome from modelling

Vaccinate boys as well!

So what about the (A level) maths?

Integration

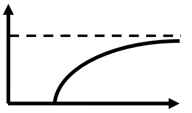
$$J(\boldsymbol{p}) = \int_0^T C_1\, N|v(t) - v_c| + C_2 N p R_0 v(t) dt$$

Modelling

Differential equations

$$\frac{dv}{dt} \approx (R_0 - 1)v$$

Curve sketching



…and Scarlet Fever?

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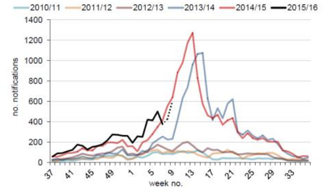

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What equation/class of equations can describe (each of) these curves?

What is the solution to $\frac{dv}{dt} = R_0(1-p)v(1-v) - v$?

What value of p minimises $C_1\, N|v(t) - v_c| + C_2 N p R_0 v(t)$?