



for ages 3 to 18+

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Realising
potential in mathematics
for all

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Making a Reuleaux triangle



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potential in mathematics
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Editors' page	2
If you can't hear, how can you do as you are told? Amy tells Rachel Gibbons how her deafness held back her education.	3
Mathematics in Unusual Places 2 Curves of Constant Width or What does a 20p coin have in common with a manhole cover? Matthew Reames continues his exploration with Year 5, 6 and 8 children providing opportunities to apply vocabulary, concepts and skills.	4
Not 'more of the basics' Jane Gabb's session on 'just doing the basics'.	9
Support and Aspiration: a new approach to special educational needs and disability Lorraine Petersen summarises the Green Paper on special education needs.	12
Illuminating Mathematics (reprinted from NASEN's Special) Pearl Barnes demonstrates how using models and images can enhance mathematical learning	14
Show Me, Don't Tell Me! Part 1 Stewart Fowlie outlines some ways in which children themselves move and act to demonstrate mathematical concepts with a few concrete equipment. Try some and send us some pictures or sketches.	19
Pages from the Recent Past Special Needs? Learning Difficulties? What can schools do about the kind Chris Mullin describes below? The importance of home with thanks to Chris Mullin, Richard Wilkinson and Kate Pickett	21
School Nightmare? His Dream School may have shattered Jamie Oliver's dreams but Rachel Gibbons suggests it has much to teach us.	22
Bus Stop Transport for London's Bus Survey	22
Pizza Club! Mundher Adhami explores different approaches to division – building concepts of 'sharing' and 'grouping'.	24
The Spirit Level: Why Equality is Best for Everyone Book Review by Jane Gabb	25

Editors' Page

It is inevitable in a system riddled with testing and league tables that the other 'half our future' (those in the lower half of the achievement range) will suffer. They are not going to gain any points for their schools to avoid the label of "failing". Indeed, it would appear that these pupils are often allocated the least effective teachers when they need the most skilled. Recently the *TES* reported "target intervention groups", where those pupils who with a little more help might reach the A* to C category were given the best teachers and extra facilities. What hope in such an atmosphere have those whose achievements are at a lower level? We would suggest that the research which is really needed is a study of what SATs etc. have done to this set of pupils.

It is interesting to look back to the time when the Inner London Education Authority abandoned "the comparability test". This was a test given to all children in their final year in primary school to give some idea of the achievement spread in each primary school to help in getting a proper spread of achievement in the comprehensive schools' intakes. The primary schools were not judged on the results of these tests but teachers still taught to the test. The ILEA therefore abandoned it and found other means of making its assessments. Lessons can surely be learnt from this

Education and a variety of learning difficulties have had a good airing in the media recently which should be useful to all of us who are keen to improve learning conditions for all young people and especially for those who are most in need of encouragement and support, the half our future who clearly have little hope of achieving results in the A* to C range.

It is more difficult today than in earlier times for groups of teachers to get out of school together to places where they have the space and the opportunity, through

discussion and the creation of teaching materials together, to improve their skills. In-school professional development is strongly encouraged but even the largest comprehensive does not provide a wide - or deep - enough background for the breadth of study that is needed. This is a serious lack because facilitating learning is not an easy task and you certainly cannot understand much about it in a preliminary period of teacher training, whether at university or in the school setting. Perhaps all skills are best learned on the job, the skills of providing rich learning environments being no exception.

Our aim in *Equals* is to provide some cross fertilisation of ideas on how to provide better learning conditions for those pupils who are not so keen to learn. But the best way we can do that is to give space to those who have excited their pupils and helped them to achieve. We do this by presenting the ideas of those of you out there who are willing to share your successes in the classroom.

It may also sometimes be useful to share failures. Jamie Oliver's dream school has shown us plenty of those from which we can learn to avoid many mistakes. And in the following pages we hope our readers will find examples which both inspire and warn. The examples include pieces of mathematics not included in the official diet because we believe that, although our pupils need an underlying steady, well planned journey through mathematics they should be encouraged, where they get hooked, to go down some of the by-ways that are not on the national curriculum map of mathematics.

So let them experiment and above all let them have fun. Present them with puzzles and challenges being sure they recognise that you believe they are capable of meeting those challenges.

If you can't hear, how can you do as you are told?

Amy¹ tells Rachel Gibbons how her deafness held back her education

Amy is a carer and a good one at that. She notices what needs doing without having to be told. She is concerned for her patients' well-being, gently jokes them out of their worries and is deft at making them as comfortable as possible.

Whenever she has a spare moment she has a book in her hand and is lost in it. Yet until she was seven years old nobody noticed Amy's hearing difficulties and by that time she had learnt to make up her own mind about what she should do

in lessons. Much of it differed from the teacher's plans for the class but that did not worry Amy. In response to being ignored she went her own way. Gradually she had persuaded her mother to buy her 5 tape recorders. In class she could hide the earphones under her long hair, thread the leads down inside her sweater to the tape recorder in her pocket and listen to music. When her tape recorder was confiscated in one lesson she had another ready for the next. At

the end of the day she went round to her various teachers collecting all her equipment ready for the next day. The intelligence shown in all these ploys was clearly not minimal, indeed I would argue that Amy was giving herself practical

lessons in logic when planning all these strategies to get her own way.

Eventually when her special needs were recognised Amy was offered a support assistant but refused because she did not want to stick out as the one person in the class

who was different and who needed extra help.

By the time she was 17 and in college she still could not read. She had got by so far, making her way in the

world by asking passers by what words meant when it was essential to know. Often Amy says that the people she asked were unable to answer her questions either. How many of them, she wonders, could not read?

Amy was popular at college and she was invited to go on holiday with some of her new friends. They told her to be sure to bring a book. Of course not being able to read, she did not possess any books. But, having seen his film, she did know something about Stephen King and she could recognise his name, indeed she had probably picked up more words than she was aware of, and she bought all King's books she could find and took them

with her. On that holiday she started to read for enjoyment and enlightenment and has never looked back.

Now, at the age of 29, Amy is an avid reader and wherever she goes she is seldom without a book in case she has time for

a few minute's delight between jobs.

Amy was offered a support assistant but refused because she did not want to stick out as the one person in the class who was different

Often Amy says that the people she asked were unable to answer her questions either. How many of them, she wonders, could not read?

Rachel Gibbons was formerly an inspector for mathematics in the Inner London Education Authority

1. Name changed to preserve anonymity

Mathematics in Unusual Places 2

Curves of Constant Width

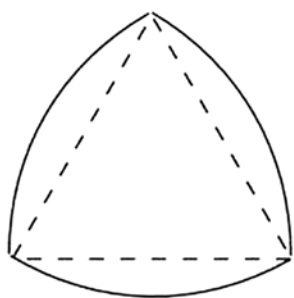
or

What does a 20p coin have in common with a manhole cover?

Matthew Reames continues his description of some work he did with Year 5, 6 and 8 children exploring the attributes of curves of constant width. This context provided opportunities for the children to apply vocabulary, concepts and skills.

Examine a Reuleaux triangle

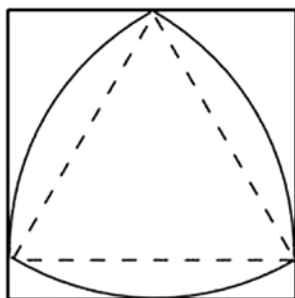
Now it is time to look at a different shape called a Reuleaux triangle. It looks like this:



As with the earlier diagrams, asking children what they notice can help bring out some important ideas. The triangle inside the curved shape is an equilateral triangle – making this an

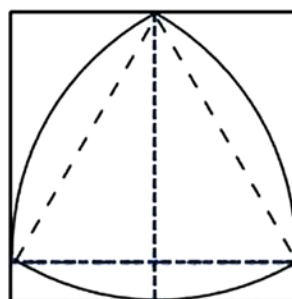
excellent time to discuss its properties: sides of equal length and equal angles. The triangle sits inside the curved shape and its vertices coincide with the vertices of the curved shape.

Now, like the circle before, we add a square.



Children might notice that the curved shape fits exactly inside the square and touches each edge once.

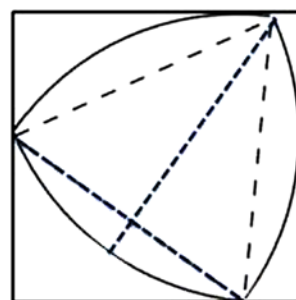
This is where it starts to get interesting! Refer back to the earlier discussion and ask how we might measure the widest parts of the curved shape? How are they



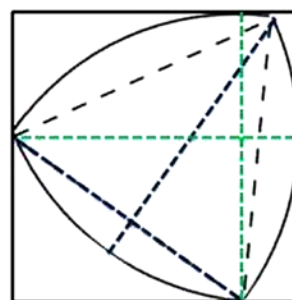
related to where the curved shape touches the square? Draw them in.

What do you notice about the lines that measure the widest parts? How might you use the words parallel or perpendicular to describe them?

Now, as we did with the circle, we will rotate the shape.



How is this diagram the same as the previous one? How is it different? Does the curved shape still touch the edges of the square? Does it touch at the same places? Can we still draw lines that measure the widest parts?

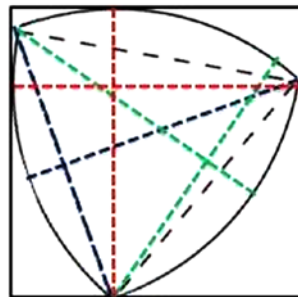
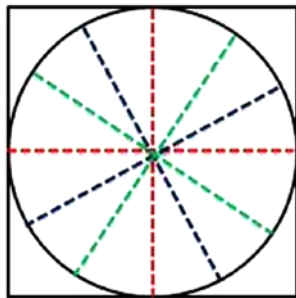
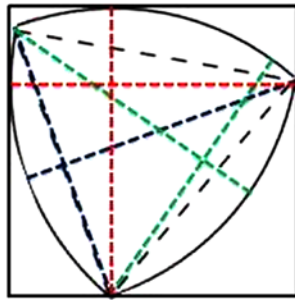


How do this new pair of lines differ from the first? How are they the same? How does these pairs of lines compare to those on the circle?

Let's rotate the curved shape once more inside the square

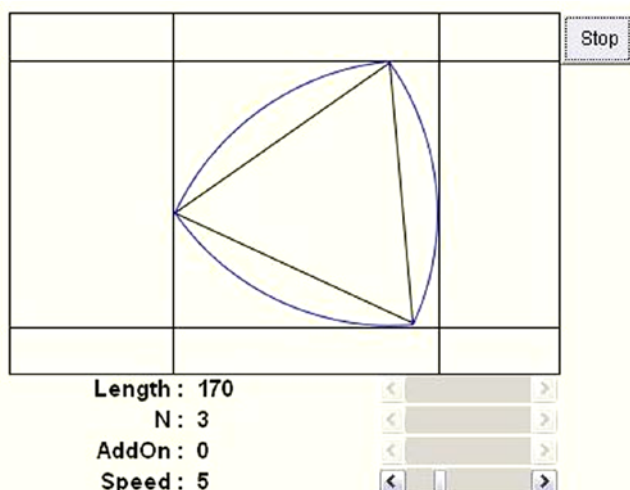
and draw in the lines that measure the width.

We can now compare this curved shape and its pairs of lines with the circle and its lines.



There are such interesting things for children to notice about these two diagrams! Asking children to compare and contrast the diagrams is a good way to start. Even though both shapes rotated inside the square, the circle always touched the square at the same place while the curved shape touched at different places. The pairs of lines that measured the width were always perpendicular to each other but the pairs of lines intersected at the same place on the circle but not the curved shape.

There is an excellent Java applet at <http://www.cut-the-knot.org/Curriculum/Geometry/CWStar.shtml> that will allow you to animate the rotation of the curved shape. The four settings allow you to investigate different sizes, speeds and shapes.



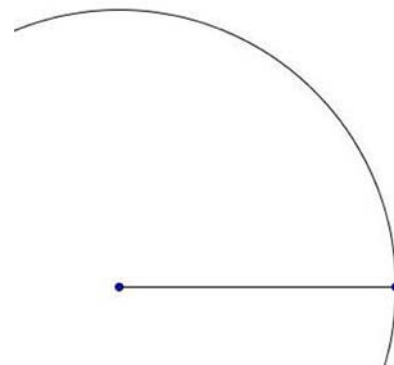
Making a Reuleaux triangle

Just as children could draw a circle and rotate it inside their L-shaped frames, they can also make and experiment with this curved shape. This is a good opportunity for children to practise constructions with a ruler and a pair of compasses. Rather than learning this skill in isolation, they are learning it in the context of an interesting investigation.

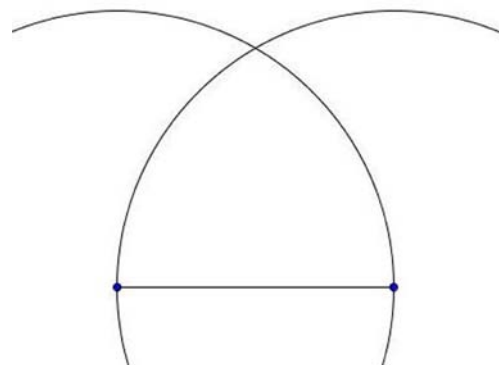
To construct this shape, use a ruler to draw a straight line on a sheet of paper. We used 10cm (it makes a shape of a reasonable size without having huge worries about it not fitting on the paper if the line is not in the exactly correct location) but other sizes will work as well.



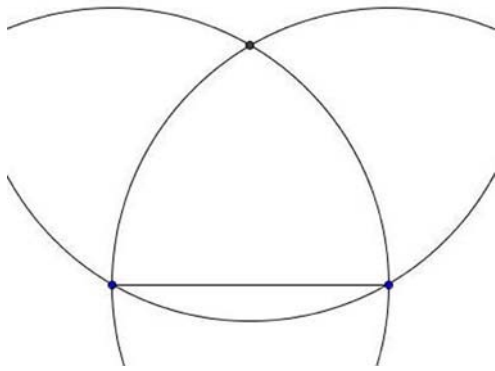
Using compasses, from one point, draw an arc with a radius of 10cm (or whatever the length of your line).



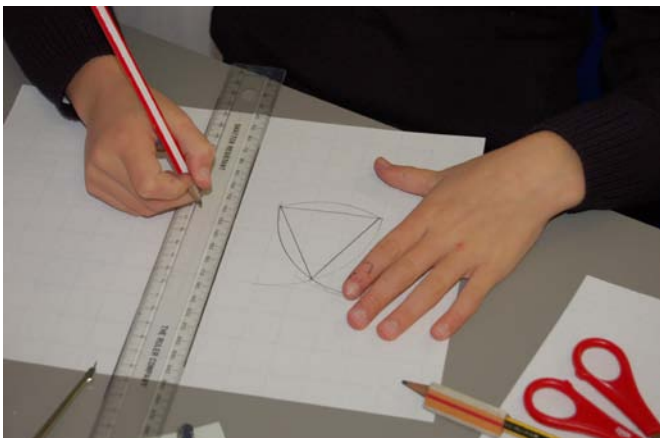
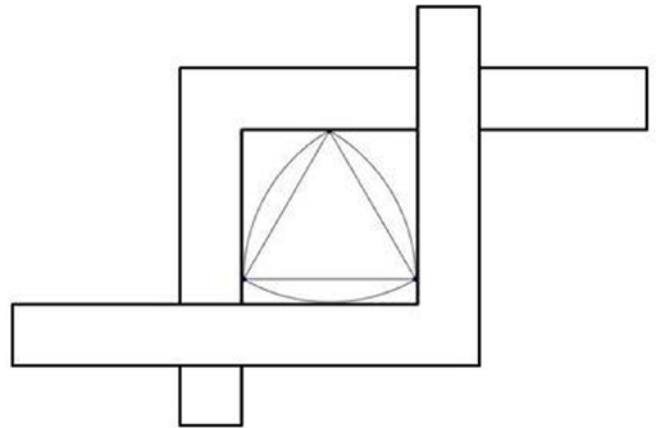
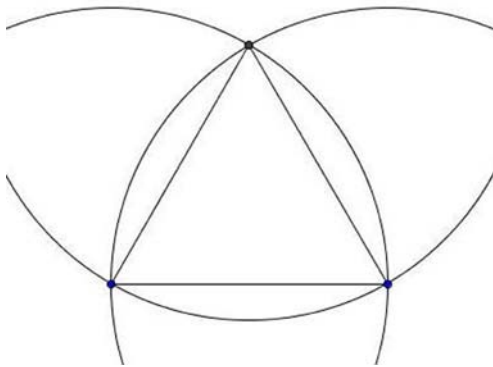
From the other point, draw another arc with the same radius.



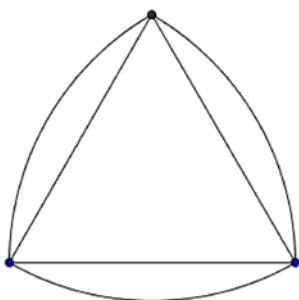
Using the point where the two arcs intersect as center, draw another arc with the same radius.



Now you should be able to see the curved shape. Children can draw in the triangle by connecting the remaining vertices.



By rubbing out the portions of the arc that are not needed, the final shape appears.



The children can now cut out the shape and use it along with their L-shaped frames to investigate how well it rotates inside the square.



Even curvier curves of constant width

The Java applet mentioned above can be used to explore curves of constant width that have more than

three curved sides. By moving the 'N' slider to the right, the number of sides increases from 3 to 5 to 7. While the 3-sided curve has a triangle inside, the other curves have star shapes. Quick use of a marker on the whiteboard can connect the points and turn a five-pointed star into a regular pentagon, a seven-pointed star into a heptagon, etc. This is an excellent opportunity to practise the names of polygons! Clicking and dragging the points of the star can alter the shape of the curve as well as the polygon resulting from the connected points. This gave us a chance to discuss the differences between regular and irregular polygons.

Some of my pupils realised that they were saying pentagon to refer to the straight-sided shape but were not sure if they could say that the curved shape was a pentagon. This was a good chance to talk about the characteristics of polygons (a closed figure with straight sides) and then ask the children to decide if the curved shape was a polygon or not.

At this point, I asked the children to predict the number of sides on the next curve of constant width. After some discussion, they decided that the next value was either 9 or 11, depending on whether the curves follow a prime-number pattern or just an odd-number pattern. One child said that he was certain it was not a prime-number pattern since a circle is a curve of constant width and a circle has only one curved side and one is not a prime number.

Moving the slider a bit more to the right showed that the next curve did indeed have nine sides. In fact, the applet will allow investigation of curves with as many as 21 sides. Several of the children noticed that as the number of sides increased, the shape got more and more like a circle. This prompted the question from one of them, 'So does a circle only have one curved side or does it have

an infinite number of curved sides?'

Some further questions my pupils wanted to investigate are:

Why does it only work with odd numbers of sides? To help answer this, several of my Year 5 students decided to create a four-sided 'bulging square' (similar to the 'bulging triangle' of the Reuleaux triangle). This alone required some rather sophisticated thought followed by experimentation with compasses and rulers.

Do the curved edges have to be the same length? As mentioned above, this can be investigated using the Java applet.

Do the shapes have to be convex? Some experimenting with a few modified Reuleaux triangles (with concave bits were removed from the curves) and their L-shaped frames helped answer this.

That's all very interesting, but so what?

An interesting property of curves of constant width was first proved by Joseph Emile Barbier in the mid-1800s. According to Barbier's Theorem, all curves of constant width that have width w have the perimeter πw . Therefore, the curve we made above (with a width of 10cm) has the same perimeter as a circle with a diameter of 10cm. A challenge for more advanced pupils would be to compare the areas of the two shapes.

In their book *How round is your circle?* (<http://www.howround.com/>), the authors investigate a number of mathematical concepts that have good engineering uses. Box-beater cams, rotary car engines and drills that drill square holes are just some of the interesting uses they describe.

This prompted the question from one of them, 'So does a circle only have one curved side or does it have an infinite number of curved sides?'

One slightly easier to demonstrate application is using a curve of constant width as a roller. Children will be familiar with lining up round pencils or felt-tip pens parallel to one another on a desk and placing a book on top. When pushed, the book will roll easily. A roller that has a cross-section that is a curve of constant width will do the same thing. To try this myself, I blu-tacked £6 worth of 20p coins together to make two rollers of 15 coins each. Unfortunately, the exactness and precision required to allow the rollers to move smoothly was not possible with my blu-tacking skills. (Bryant and Sangwin do show an example of a Reuleaux triangle-shaped roller on their site.)



You can still demonstrate the rolling properties of these curves by using several 20p or 50p coins and two rulers. Place the rulers parallel to one another with several 20p or 50p coins in between (or, if you have any, some 2 Kč or 20 Kč coins from the Czech Republic). Hold one ruler still while moving the other back and forth across the line of coins. It should roll as smoothly as if you used circular coins!

Though I am not an expert on the internal operations of vending machines, I have been told that one benefit of 20p and 50p coins being curves of constant width is that vending machines can easily verify the width of the coins from any angle. A question for students may be, 'Would the Australian 50 cent coin or the 2 dollar coins and 20 cent coins from Hong Kong mentioned earlier work the same way? Why or why not?'

Finally, curves of constant width work in three-dimensions as well. Solids of constant width are possible and are fun for children to investigate. Bryant and Sangwin include

several videos on the site above and, on occasion, may be able to provide a set of three of these solids to interested groups. A book placed on top of these solids will move just as if they were on three ball-bearings. The children loved being able to get down at table-top level and watch the solids rotate as the book moved.

As I mentioned at the start of the article, investigating curves of constant width provided an excellent opportunity for my pupils to take part in mathematical discussion and investigation. As far as I know, the heading 'Curves of constant width' has yet to appear on any primary mathematics curriculum. What my pupils discovered, however, was that they had to use and understand a huge number of concepts during their investigating. When they had questions or encountered something new, they were far more receptive to learning a new term, concept or skill when it was in this context than if they had learnt it in isolation.

All this brings me back to the subtitle of the article: What does a 20p coin have in common with a manhole cover?

A question, supposedly asked by Microsoft in employment interviews, is, 'What shapes can you make a manhole cover so that it cannot fall through the hole?'

The answer is, 'A shape that is a curve of constant width.'

Further reading:

http://www.cut-the-knot.org/do_you_know/cwidth.shtml
has good questions to ponder

http://en.wikipedia.org/wiki/Reuleaux_triangle

<http://mathworld.wolfram.com/CurveofConstantWidth.html>

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Not ‘more of the basics’

Jane Gabb’s session at this year’s Mathematical Association Conference was inspired by a number of teachers over the years who when speaking of their pupils who struggled with mathematics said, ‘We just do the basics’.

This means in practice calculation, and in fact practising calculations, which generally leads to a very dry uninspiring curriculum which fails to motivate the very youngsters who need motivating. It also reinforces failure on a regular basis, as those who struggle with mathematics almost always are weakest in number and calculation.

My starting point is that pupils who struggle with mathematics:

- don’t enjoy mathematics lessons
- lack confidence in their ability to do mathematics which hampers their learning
- are weakest in number and calculation
- find abstract concepts very difficult

What then do they need in order to progress?

- small successes – answering questions, solving problems
- visual representation and practical equipment to manipulate
- to begin to understand the connections between aspects of mathematics
- a wide mathematics curriculum offer – not just number
- experience of working collaboratively
- opportunities to use mathematics that feel relevant to them, but not just tedious real life contexts!

What don’t they need?

- more practice at things they **can’t do**
- someone doing mathematics for them so that they can **complete tasks**

- being in a classroom where everyone struggles with mathematics, (unless they have an extraordinarily gifted teacher!)
- lots of written work

By addressing the items in the ‘what do they need’ list, those in the first list can be addressed without recourse to those in the ‘not needed’ list. Some examples follow.

Small successes

Most pupils who see themselves as ‘no good at mathematics’ lack the confidence to put up their hands to answer a question. If they are to have success then the first thing that should be implemented is a ‘no hands up’ policy combined with a classroom ethos where it is okay to make mistakes (because they can be used to help everyone’s understanding). Then some random way of choosing pupils to answer (lolly sticks with names on is one) can be used. This will be resisted by the pupils to begin with (especially those who usually answer the questions) but they all need to see that it is a much fairer system where everyone gets to ‘have a go’ and therefore the opportunity to be active and to learn.

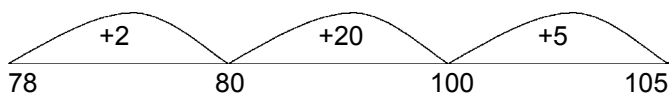
Visual representations and practical equipment

Too often these are seen as Key Stage 1 accompaniments to mathematics, rather than as support throughout primary and secondary school for pupils who would benefit from a more concrete approach. Too many children begin to fail in early Key Stage 2 because they are expected to be able to deal with more abstract concepts than they are ready for. (See final bullet in first list)

Some examples:

- Blank number lines for subtraction – adding on strategy. Many pupils 'don't get' vertical subtraction (especially decomposition) but can work well with a number line strategy. E.g. $105 - 78$

Typically: 105 (This is what I call lazy subtraction – 5 subtract 8 is too hard, so I'll do $8 - 5$ instead')

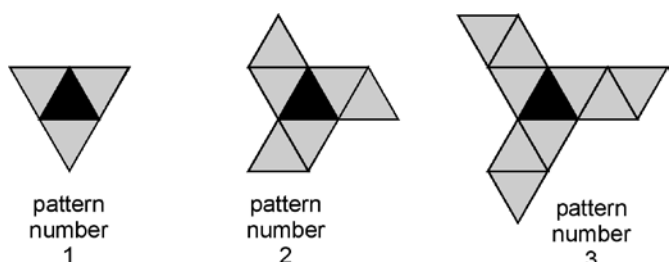
$$\begin{array}{r} 105 \\ - 78 \\ \hline 173 \end{array}$$


So $105 - 78$ is $2 + 20 + 5 = 27$

- Arrays to make the connection between multiplication and division

* * * *	$3 \times 4 = 12$
* * * *	$4 \times 3 = 12$
* * * *	$12 \div 3 = 4$ (How many 3s are in 12?)
* * * *	$12 \div 4 = 3$ (How many 4s are in 12?)

- Encourage the use of colour for emphasis in exercise books e.g. draw a red box round things that need to be remembered
- When doing sequences, use visual patterns and relate the term-to-term relationship to the context e.g.



Number of tiles in pattern n is $3n + 1$. **Each arm has n grey triangles and there is always the black one in the middle**

Make a table and then draw the relationship on a graph to explore what gradient and intercept mean in a context.

Practical equipment to manipulate

- For ordering anything – pieces of paper to move around
- Base 10 apparatus to show what place value means
- Real (or plastic) coins for work with money
- Jigsaws and matching activities (Free software to produce these easily at: <http://www.mathsnet.net/jigsaw/index.html>) These can be used by pupils working in pairs and can be tailored to your pupils' needs. They will do a lot more mathematics than they would if these were in a book or on a worksheet!
- Strips of paper for fractions – folding to make halves, quarters etc and comparing with another strip for thirds, sixths etc. This can lead to the use of a fraction wall for finding equivalents and 'seeing' how fractions compare.
- An A4 sheet divided into 12 (3×4 array) can be used to find equivalent fractions (in $1/12$ s and $1/6$ s) to $1/2$, $1/3$, $1/4$ etc.
- 3D shapes to handle when discussing properties
- Shapes to fold for reflective symmetry
- Shapes to tessellate - irregular triangles and quadrilaterals as well as regular shapes

Connections

Pupils who struggle with mathematics often see it as a set of unconnected 'facts' that they have to learn. This gives them an impossible task.

Typically they don't know:

- that multiplying and dividing by 2 are the same as doubling and halving
- that knowing their number bonds to 10 can help them

to do many calculations without always counting on their fingers

- the connections between multiplying by 3, 30 and 300
- the relationships between addition and subtraction, and multiplication and division
- that factors and multiplication tables are connected
- that the 3 times and 6 times tables are connected through a doubling relationship

A wide mathematical curriculum

Imagine how you would feel about mathematics if the only bit you met was calculation which you couldn't get the hang of. No wonder those who struggle turn off the subject.

Many pupils who are seen (by their teachers and themselves) as no good at mathematics, can solve practical problems, can think things through, can interpret graphical information, can work at a much higher level with shape and space and measurement.

It is possible to address lots of 'basic' mathematics through these other aspects. For instance:

- Measurement as a vehicle for working with multiplying and dividing by 10, 100 when converting units
- The angle sum in triangles as a way of practising addition and subtraction, and complements
- Thinking mathematics lessons (e.g. Cognitive Acceleration in Mathematics Education – CAME and the Let's think through maths series) Teachers are often amazed at how well their 'less able' pupils do with these thinking activities

Any of these can be more motivating than endless practice, and can lead to more confidence in tackling mathematical activities.

Working collaboratively

Many times when I have visited a 'bottom set' the pupils are seated as far apart as possible, in order to keep control and stop the pupils interfering or distracting each other. Introducing pair work or group tasks into such a situation is demanding, but can be very rewarding. Practical tasks such as jigsaws and card sorts provide a useful first step and can help persuade reluctant teachers and pupils that working together can be a fruitful mathematical activity. The software outlined above can be used to create tasks which are tailored for a particular group's needs and pupils will do far more 'work' if tasks are presented like this than they would do if given a text book or worksheet with the same exercises.

Group problem-solving could be a next step, using tasks which have been developed for this purpose. For more on this, including some examples of tasks, go to: <http://nrich.maths.org/2547>. This is an article about co-operative problem-solving through what is called the 'pieces of the puzzle' approach, where individual pupils have a piece of the puzzle and need to work with the rest of the group to solve it.

CAME activities are also useful for exploring pair and group work.

Relevant mathematics

Relevant mathematics is not the same as 'real life' mathematics, especially as presented in many text books. The key is 'What is relevant to these students?', not what the teacher might think is important for their future life. Finding out what students are interested in and weaving a mathematics curriculum around that is a challenge for most teachers. However, the rewards, i.e. students who are motivated to work at something, are very great. Of

course, not everything can be made relevant, but if it is clear that the teacher is listening to students and is interested in what makes them tick, the relationship will be very different and this is a good start. Teaching and learning are primarily about this relationship; without a meeting point of this kind, the classroom is a sterile and unrewarding place for both teachers and learners.

Support in the classroom

Many pupils who struggle are assigned to a teaching assistant, and rarely have teaching from a teacher. Many pupils and teaching assistants (and teachers) are more interested in task completion than understanding and learning mathematics. This situation leads to pupils being 'led' towards 'the answer' so that they can complete the 'work'. It has little to do with learning or motivation, in fact it reinforces dependency and the feeling that 'I can't do this unless I have help'.

Conclusion

There are too many ideas here to implement immediately, but you are invited to choose something which you could action tomorrow, and try it out in the spirit of experimentation. You may not achieve immediate success because transformation takes time, but you will be on the road to a more beneficial situation than the one outlined at the beginning of this article. Anything you try could be the subject of a future article for *Equals*, so please write and tell us of your experiences. We are always interested in publishing work from teachers in the classroom.

Jane Gabb is retired mathematics advisor and works as a 1-1 tutor in Winston Churchill School, Woking

Support and Aspiration: a new approach to special educational needs and disability

Lorraine Petersen supplies notes on the recent Green Paper.

Summary Document

The Green Paper published on March 9th offers the government's proposals for the future of special educational needs and disability. Nasen welcomes the report and is very pleased that the government and Officers of the Department for Education, have committed a great deal of time and energy to ensuring a robust and fair process, taking due regard of all the responses they received during the consultation last year.

Chapter 1 : Early Identification and support

Early identification of need

- Health and development review at 2/2.5 years
- Support in early years from health professionals
- Greater capacity from health visiting services
- Accessible and high quality early years provision
- DfE and DfH joint policy statement on the early years
- Tickell Review of EYFS
- Free entitlement 15 hours for disadvantaged 2 year olds
- A new approach to statutory assessment
- Education, Health and Care plan to replace statement
- A more efficient statutory assessment process
- DoH improve the provision and timeliness of health advice

Reduce time limit for current statutory assessment process to 20 weeks

Chapter 2 : Giving parents control

Supporting families through the system

- Continuation of Early Support resources

Clearer information for parents

- Local Authorities to set out local offer of support
- Slim down requirements on schools to publish SEN information

Giving parents more control over support and funding for their child

- Individual budget by 2014 for all those with EHC plan

A clear choice of school

- Parents will have rights to express a preference for a state-funded school

Short breaks for carers and children

- Continue to invest in short breaks

Mediation to resolve disagreements

- Use of mediation before a parent can register an appeal with the Tribunal

Preference for a state-funded school

Chapter 3 : Learning and achieving

Developing excellent teaching practice for SEN in schools and colleges

- Initial funding for ITT – increase placements in Special School
- Outstanding Special Schools apply to be teaching schools
- Continued funding for SENCO award

Effective leadership is critical to changing ethos and approach in schools and colleges

Getting the best from all school and college staff

- Improve SEN training for those working in colleges

The Achievement for All approach

- Achievement for all developed across country

Challenging low expectations of, and targeting support for, children with SEN

- Every Child a Reader and Every Child Counts transition funding

- Phonics based training
- Replace school action and school plus with single school based SEN category

Identifying and tackling the causes of difficult behaviour

- Anti Bullying Alliance share good practice
- Trial of new exclusions system
- Improving access to wider behaviour support

Special Schools Special schools become academies

Special Free Schools

Stronger school accountability

Chapter 4 : Preparing for adulthood

Planning for young people's futures

A broad range of education and learning opportunities

- Wolf Review

Employment opportunities and support

- Role of Disability Employment Advisers

A coordinated transition to adult health services

- Joint working across all services

Support for independent Living

Chapter 5: Services working together for families

Local authorities and local health services will play a pivotal role in delivering change for children, young people and families

Reducing bureaucratic burdens on professionals

Empowering local professionals to develop collaborative, innovative and high quality services

Supporting the development of high quality speech and language therapy workforce and educational psychology profession

Encouraging greater collaboration between local areas

Extending local freedom and flexibility over the use of funding

Enabling the voluntary and community sector to take on a greater role in delivering services

Exploring a national banded funding framework

Bringing about greater alignment of pre 16 and post 16 funding arrangement

Lorraine Petersen is Chief Executive Officer for Nasen

Illuminating Mathematics (reprinted from NASEN's *Special*)

Pearl Barnes demonstrates how using models and images can enhance mathematical learning

Over the years a number of mathematical resources have become fashionable and subsequently faded from memory, only to be discovered covered in dust lying at the back of the cupboard! But why were such games developed in the first place and do they still have a place in the teaching of mathematical skills and concepts today?



Multi-sensory learning approaches

Children learn in a number of ways through a variety of media which integrate a number of senses. There has been an abundance of research into multi-sensory approaches to learning, and educationalists are universally familiar with the 'VAK' – visual/aural/kinaesthetic – approaches to learning. Multi-sensory approaches are maintained as they enable children to utilise an array of senses to facilitate and embed the learning. Some children are predominantly visual learners and will benefit from a largely visual approach, whereas others are kinaesthetic learners and learn well through handling and manipulating resources, while others have a strong auditory memory and are able to process verbal information successfully. However, a

Resources should be visibly utilised within the educational setting, but transparent in their use

combination of approaches is demonstrably the most effective strategy when differentiating teaching to meet a range of learning styles within a single class. Resources and games which are interactive and engaging are now used routinely within well-differentiated mainstream classroom settings. But is there a more effective way to enhance the learning of children who are continuing to struggle to understand and grasp the key concepts?

Building knowledge or just having fun?

I have, in previous articles, endorsed the importance of children engaging mentally with resources, as opposed to merely going through the motions and playing an enjoyable game. The importance is in inwardly constructing appropriate meaning of the key concepts from engaging in the activities as opposed to passively going through the routine of the game or activity, missing

the purpose of the activity and failing to engage with the concept it is designed to enhance. Research has now shown that without careful planning and facilitation,

resources and activities may become a barrier in their own right to learning, detracting from the key learning objective which was the original object of the activity. Using the resource appropriately in itself can become the objective of the activity, diverting from enhancing mathematical learning. The tendency is to focus on what children do (process or procedure) as opposed to what children are thinking, and internalised action. How, therefore, can a teacher ensure that a child is constructing the meaning and learning intention which the resource and activity are designed to promote?

Using resources within fun and enjoyable games does not necessarily guarantee the acquisition of conceptual understanding, and I must emphasise that in order for successful mathematical learning to occur, resources should be visibly utilised within the educational setting, but transparent in their use – not masking the mathematical learning.

The challenge of using resources to acquire conceptual understanding is therefore in enabling children to internalise the concrete representations by ensuring that the resource is the tool or carrier for enhancing mathematical learning as opposed to the objective of the learning. It is not the use of resources per se which is important for learning but how they are used.

Abstract versus concrete

Mathematics is composed of abstract concepts and principles. Consider the Arabic numerical system itself.

1 2 3 4 5 6 7 8 9 10

How do these numerals communicate or translate what they are representing? Although they are visual images, and therefore appeal to the visual memory, they are merely abstract symbols which represent quantities of things. Their shape and form tell us nothing about what they are representing – how '7' relates to '8', or whether a number is bigger or smaller than another number. The cardinal value is the unique value of the number, whereas the ordinal value is the position of the number, such as first, second, third and so on. Many children, when learning to count, think of a number as the ordinal as opposed to the cardinal. For instance, a child learning to count to say, the number 7, stops at seven. Seven becomes the last number counted – the ordinal – the seventh count. In reality, seven is the sum of all seven parts, and is composed of $1 + 1 + 1 + 1 + 1 + 1 + 1$. Table 1 below demonstrates the subtle differences between the cardinal and ordinal rules which can lead to many misconceptions regarding partitioning a number into its

constituent components, counting on and the notion of equivalence and value when one number is visualised as more or less than another number. So, there are several issues to consider:

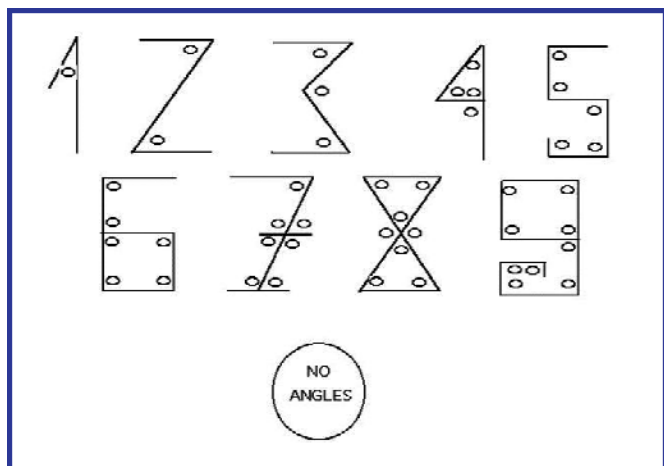
- > The representative (visual representations) Arabic number system doesn't provide any clue regarding the *cardinal value, or quantity*, of the number it is representing.
- > The visual representation also doesn't illustrate the magnitude of the value – that 4 is just 1 bigger than 3 and less than 5, whereas 9 is 8 more than 1.
- > The symbol doesn't provide any indication as to its *relationship* to other numbers.
- > Ordering and sequencing numbers orally and visually in a number line instil in children the construction of the number sequence without necessarily applying the numerical value to that sequence; it tends to encourage ordinality as opposed to cardinality – the sum of the whole set.

The patterns in the number system don't become obvious until the third decade (ie numbers over 20). The number track featured below in table 1 demonstrates four teddies. But a child counting to four may perceive the fourth teddy as the number 4 as opposed to the *whole set* of four teddies. To illustrate the 'fourness of 4' it is important that children are provided with opportunities to count different sets of objects in different ways.

Table 1 Cardinality versus ordinality

			
1	2	3	4

Take this visual representation of the Arabic numeral system:



This visual representation is more meaningful as the cardinal value is represented in the number of angles or dots embedded into each symbol.

Interestingly, the bar and gate notation is often employed, which in itself communicates more readily the value of what it is representing, but does not assign a name.

For example:



Research into mathematical development has demonstrated that if very young children are provided with a set of objects and asked to record how many objects there are, their method of recording would be similar to the bar and gate notation. But this is laborious and open to error, and although it utilises multiples of five, it doesn't lend itself easily to encourage calculating without counting. The Arabic code on the other hand, does have the benefit of being systematic; the sequences and patterns enable ease of calculation without arduous counting. In order to bring to life the meaning or cardinal value of the number system, we therefore need to dust off those resources and begin to use them within appropriate mathematical contexts.

The use of appropriate resources

When illuminating abstract mathematical concepts, resources need to be chosen carefully in order to enhance and communicate the meaning either through creating a mental image of the concept as illustrated earlier, or through manipulating resources to demonstrate the concept occurring.

Figures 1–5 below illustrate some of the key mathematical ideas for Foundation Stage and Key Stages 1 and 2. It is beyond the scope of this short article to demonstrate every mathematical concept, and indeed the various solutions to enhancing these concepts, but a useful free resource to obtain further insight can be downloaded from the DfE website (*Models and Images: Using models and images to support mathematics teaching and learning in Years 1 to 3*).

Some resources or activities are better at imparting some mathematical ideas or concepts than others. For instance, counters are great for learning to count and for counting sets, but are hopeless for encouraging counting on from a given number. Careful consideration as to the appropriate resource for the specific idea or concept needs to be given to enhance learning effectively.

Key features for using resources effectively

The need for early identification and intervention

- > Prevent the onset of mathematics anxiety and intervene while the child is still enthusiastic and willing to participate. If resources are introduced routinely at a young age, they become part of everyday mathematics and reduce the stigmatising element. Early correct use of resources enables a foundation to be developed without the need for undoing prior misunderstandings.

Personalised learning

- > Provide targeted intervention and support to motivate the child by using resources which are meaningful

and appealing to them. Identify specific areas of difficulty, misconceptions and any underlying root causes, through detailed assessment.

Multi-sensory learning

- > Children learn in a variety of different ways at different times, so use a range of approaches to ensure that all children have the maximum opportunity to learn in the way that suits them best.
- > Reduce the need for the child's reliance on weaker memories such as auditory sequential memory, short-term or working memory.



Figure 1
How many more? Addition pairs



Figure 2
Partitioning a two-digit number

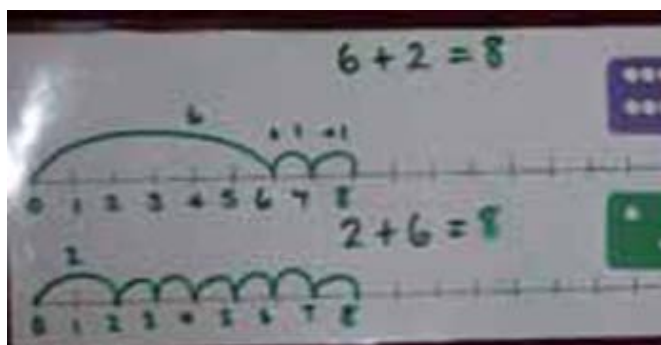


Figure 3
Commutativity in addition



Figure 4
Commutativity in multiplication

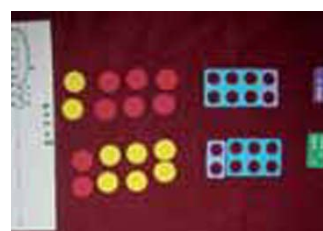


Figure 5
Equivalence

Mediated learning

- > Allow children to become reflective learners through careful guidance, where the teacher doesn't correct mistakes, but guides children to look carefully and more closely at their work, encouraging checking and discussion.

Facilitated reflective learning opportunities

- > Ask 'How did you do that?', 'Why did you choose to work in that way?' to encourage children to explain their mathematical thinking, demonstrate through resources and discuss their processes with others. By reasoning their thoughts through, children are able to construct meaning and correct mistakes independently.

Building on what a child knows and can do

- > Ensure that children are successful, and that misconceptions are not formed.

Scaffolding, nudging, guiding as opposed to direct teaching

- > Provide guidance to encourage children to explore for themselves, enabling the internalisation of meaning – resources are excellent media for encouraging exploration.

Making the objective explicit

- > Be aware that children do not always intuitively grasp the concept which the teacher expects them to grasp. Making the objective explicit and referring regularly to the objective ensure that the child's learning is focused towards the objective and not the activity or resource.
- > Be aware that children do not always see what teachers expect them to see.

Clear and unambiguous language

- > Keep language clear, distinct and consistent when using resources.

Correct use of mathematical language which is introduced in bitesized chunks

- > Ensure that vocabulary is introduced within meaningful contexts and is supported through appropriate models and images.
- > Introduce new vocabulary a little at a time and ensure complete understanding.

Over-learning, repetition

- > Revisit prior learning in order to embed the learning into the long-term memory.
- > Generalise skills and concepts through repetition of the same skill/concept utilising *different* resources/ images.

Child's familiarisation with the resources

- > Use resources consistently to ensure familiarity.
- > Use familiar resources to introduce a new mathematical skill or concept and ensure that the child is at ease with the equipment and able to focus on the new skill.

Child's motivation

- > Maths anxiety can be a barrier to learning, so encourage motivation by using engaging resources which stimulate the child and distract the child from any anxiety.

Subject knowledge

- > Use the deep subject knowledge of the teacher and ability to match the resource to the mathematical idea.

Appropriate and achievable challenges which take the child beyond the prior learning

- > Set challenges which stretch the learner little by little to move them on in their learning more effectively than setting challenges which take longer to achieve.
- > Move learners away from the iconic representations to the more abstract concepts when they are ready.

Availability of resources/mathematics-rich learning environment

- > Have a range of resources readily available to provide the best learning environment for children.
- > Provide a stimulating learning environment with colourful and interactive displays of the children's recording.

Consistency yet flexibility

- > Ensure consistency in approach, language use and use of resources – essential for effective learning.
- > Flexibility is needed to 'go with the flow' and follow the children's train of thought.

On the spot responses through active listening

- > The ability of the teacher to respond spontaneously and instantly to the child's cognitive processes and move the child on at their pace is the essence of child-centred learning, requiring an almost intuitive response to the child's inner thought processes and functions.
- > Active listening is the means whereby a teacher can respond appropriately to the child.

Key message

Resources in themselves have no mystical quality for enabling children to learn through them. It is how resources are used, alongside modelling, use of correct vocabulary and mathematical language and matching the resource with the concept to be developed which provides the platform for learning. So let us dig deep within our cupboards, dust off those once discarded resources and create active learning opportunities.

Pearl Barnes is a training and assessment consultant and a specialist in mathematics as well as learning difficulties.

Show Me, Don't Tell Me!

Part 1

Stewart Fowlie outlines some ways in which children themselves move and act to demonstrate mathematical concepts with concrete equipment. Try some and send us some pictures or sketches.

Today a teacher of a reception class can assume that all her pupils have attended a nursery class and have some idea of counting. To make sure they understand her and her meaning she might get a boy and a girl to stand beside her and tell them that there are the same number of boys as girls. Note that 'same' rather than 'equal' should be used. Up to 5 pairs altogether should join the teacher. After each change the teacher should say that there are still the same number of boys and girls. She should then arrange the children so all the boys are standing on one side of her and all the girls on the other. Looking at the rows of boys and the rows of girls it is easy to see there are the same number of each and what the number is. If a boy and girl go and sit down there will still be the same number of boys as girls.

If a boy joins there will be more boys than girls.

If a boy leaves there will be fewer boys than girls.

If a girl joins there will be more girls than boys.

If a girl leaves there will be fewer girls than boys.

The teacher should start using her fingers on one hand to stand for boys, and on the other for girls going up to 10 children in all. Children should practise doing the same and also draw the fingers:

I standing for each finger,

V for a hand of 5 fingers,

X for 2 hands or 10 fingers (a thumb is to count as a finger!)

I II III IIII V VI VII VIII VIII X

Then 1, 2, 5, 10 standing for 1p, 2p, 5p, 10p coins.

1 2 21 22 5 51 52 521 522 10

Finally 1 stands for a playing card,

2 3 . . . 10 for a pile of that number of cards.

1 2 3 4 5 6 7 8 9 10

Children should practise listing - one below another - different families with 1, 2, 3, 4 children.

Having seen this, children can see how to list families of any size.

Next list different families with 2 children (order of birth matters), and with 3 children.

to list even the different families with 4 children is a task more suited to a 12 year old.

1	2	3	4
		BBB	BBBB
	BB		BBBG
		BBG	BBGG
B			BBGB
		BGG	BGGG
	BG		BGGB
		BGB	BGBB
			BGBG
			GGGG
		GGG	GGGB
	GG		GGBB
		GGB	GGBG
G			GBBB
		GBB	GBBG
	GB		GBGB
		GBG	GBGG

Now we begin working numbers out: each pupil should

be given a suit of 13 cards - each card to represent a child.

In a class there 6 boys and 4 girls. How many children are there in the class?

First the teacher can get 6 boys to stand in a row and then add 4 girls one by one to the end of the row as she says: "7, 8, 9, 10."

Using a card to represent a child, the pupils make piles of 6 cards and 4 cards (6 boys and 4 girls). They move the cards in the smaller pile to the larger pile, saying as they move them:

7,8,9,10.

"10 is called the Sum of 6 and 4."

A 5 year old boy was told that there will be 5 children at his party (CCCCC) and 3 of them will be boys (BBBCC). He happily said there would be more boys than children at the party. This mistake is normal at mental age 5.

What is the difference between the number of boys and the number of girls?

6, 4 5, 3 4, 2 3, 1 2, 0

At each stage a boy and a girl are removed, so the excess of boys is unchanged.

"2 is called the *difference* between 6 and 4."

Next, using the same process we used to find the sum of 6 and 4, find all the pairs of numbers having sum 10:

B 10 9 8 7 6 5 4 3 2 1 0

G 0 1 2 3 4 5 6 7 1 0 10

Using the same process, we find pairs of numbers differing by 2:

B 2 3 4 5 6 7 ...

G 0 1 2 3 4 5 ... and so on for ever.

From the 2 lists we see that the one pair of numbers with a sum of 10 and a difference of 2 is, clearly 6 and 4.

In the next issue, Stewart Fowlie will suggest some ways of making multiplication, factorization and ratio more understandable.

Stewart Fowlie is a tutor of children with learning difficulties in Edinburgh

What do you make of all this?

Wildlife at Hope Farm

Grey partridges have gone from 0 to 4 pairs

Linnets have gone from 6 to 30 pairs

Yellowhammers have gone from 14 to 36 pairs

Skylarks have gone from 10 to 41 pairs

The floristic diversity of field margins has increased

103 species in 2000

168 species in 2009

Insects and mammals:

25 species of butterfly have been recorded since 2000 of which

20 are regularly seen during surveys

230 macro moth species

4 species of bat and

6 species of small mammals have been recorded

With thanks to the RSPB

Special Needs? Learning Difficulties?

What can schools do about the kind Chris Mullin describes below?

Sunday 8 April 2001

Sunderland

Sunshine. With the children to fly their kite and play football. Later we spent a couple of hours on the beach at South Shields, which was fairly crowded. After we had been there a while, two rough looking women with seven young children, one a baby, in tow turned up and sat on the concrete slipway. One of them changed the baby's nappy which she handed to a small boy who simply tossed it into the sea. She then handed him the tissues with which she had been wiping the baby's bottom and he tossed these into the air scattering them over the slipway. Neither of the women batted an eyelid. What hope for the children? (p193)

Friday 29 June 2001

Sunderland

I walked through the town centre. Sinister, truanting youths lurked in shop doorways. Pasty, vacant faces reflecting empty lives. It is hard not to feel sorry for them. (p 213)

Chris Mullin (2010) *A View from the Foothills* (London, Profile Books)

A challenge we all should meet

Jane Imrie of NCETM wrote recently that the challenge in the mathematics classroom is not about introducing harder content because the more content there is the more time is spent on technique and not using mathematics, which would be like spending all your music lessons learning harder and harder scales and arpeggios but never playing a piece of music.

Kate Pickett and Richard Wilkinson start to analyse the problem

Although good schools make a difference, the biggest influence on educational attainment, how well a child performs in school and later in higher education, is family background. In a report on the nature of education in Britain, Melissa Benn and Fiona Miller describe how:

One of the biggest problems facing British schools is the gap between rich and poor, and the enormous disparity in children's home backgrounds and the social and cultural capital they bring to the educational table.

Children do better when their parents have higher incomes and more education themselves, and they do better if they come from homes where they have a place to study, where there are reference books and newspapers and where education is valued. Parental involvement in children's education is even more important.

Richard Wilkinson & Kate Pickett, (2010) *The Spirit Level: Why Equality is better for everyone* (London, Penguin Books)

See review page 25

- but, of course, it has to be solved in the classroom – any ideas?

School Nightmare?

His Dream School may have shattered Jamie Oliver's dreams but Rachel Gibbons suggests it has much to teach those working in the nation's classrooms.

This is especially true if we are trying to succeed with pupils who have little respect for learning and who may be growing up in homes where education is not valued. It should show those who have only been in classrooms as pupils themselves something of what a teacher's job is really like. And, for those of us who have chosen to take up the teaching role, it seems to me it gives us much food for thought,

Some of those taking the teacher's role were far too arrogant and showed no respect for the members of the class in front of them. David Starkey, for example, still needs a course in what I call the basics - not English and mathematics but learning how to live in a community responsibly, respectfully, reflectively, reliably and co-operatively and without self-aggrandisement. That, it appears to me, Starkey, for all his years, has

not yet learnt to do. It is by no means easy always to show respect to a group of youngsters who are being deliberately disruptive but shouldn't the teacher blame him or herself for that state of affairs? Those youngsters in front of you did not did not ask to be in your classroom (or the chances are they didn't). Why should they listen to what you want to tell them?

And it is not necessarily the most talented in a particular subject who will be able to put it across. A love of the subject is certainly needed but those to whom its study comes most easily may not see how to put it across in acceptable terms to an uninterested group.

Rachel Gibbons was an inspector for mathematics in the Inner London Education Authority

Bus Stop

Here is the real thing: a questionnaire used by Transport for London to collect information enabling them to improve the bus services in the city. How you use it in your classroom will be determined to a great extent by the age of your class. Should they be encouraged to fill the surveys out according to the last time each one of the travelled on a bus? Perhaps they might discuss how many seats there are on different types of bus, how full the bus was on which they last travelled.

It could form the basis in a primary classroom for a consideration of the area served by the school, perhaps marking the homes of the pupils on a large scale map and then considering how long it takes each one to get to school. At a later stage a class might consider bus timetables and how these might be changed to serve the community better. Do you need different numbers of buses at a certain time of the day and why?

Whatever you choose to do with it, being a real survey, just studying it should give your pupils some realisation of the value of statistics.

Transport for London (TfL) Passenger Survey - Side 1

756908

Office Use only

This survey is being carried out to help us provide a better, more tailored, bus network. Completion is voluntary and you may be surveyed on more than one bus. Please accept a card each time and complete both sides. Our surveyors can provide the use of a pen if necessary. By submitting this form, you are authorising TfL, its subsidiaries and service providers, to use your personal details for research purposes only. These details will be properly safeguarded and processed in accordance with the requirements of the Data Protection Act 1998.

Please hand the card in as you leave the bus, even if the questionnaire is not completed.

1 How did you get to where you boarded this bus?
(Please cross ☒ one box only)

another Bus ☐ 1 | London Underground ☐ 3 | DLR/Tram ☐ 5
National Rail / London Overground ☐ 2 | Walked ☐ 4 | Other ☐ 6

If by another bus, what Route No?

2 At what address did you start the journey you are making NOW? (Street number and name). (If you don't know the street number give Property Name eg Tesco)

Street No.
(or Property Name)
and Street

Locality

Town

POST CODE

3 Why were you there? (Please cross ☒ one box only)

Home ☐ 1 | Work ☐ 5
Picking up/Dropping off Someone ☐ 2 | Shopping/Personal Business ☐ 6
your Hotel/Hostel ☐ 3 | Education ☐ 7
Social/Recreation ☐ 4 | Other ☐ 8

4 How will you continue your journey after leaving this bus?
(Please cross ☒ one box only)

another Bus ☐ 1 | London Underground ☐ 3 | DLR/Tram ☐ 5
National Rail / London Overground ☐ 2 | Walk ☐ 4 | Other ☐ 6

If by another bus, what Route No?

Please turn over



Side 2

Please turn over and complete Side 1 first

5 At what address will you finish the journey you are making NOW? (Street number and name). (If you don't know the street number give Property Name eg Tesco)

Street No.
(or Property Name)
and Street

Locality

Town

POST CODE

6 Why are you going there? (Please cross ☒ one box only)

Home ☐ 1 | Work ☐ 5
Picking up/Dropping off Someone ☐ 2 | Shopping/Personal Business ☐ 6
your Hotel/Hostel ☐ 3 | Education ☐ 7
Social/Recreation ☐ 4 | Other ☐ 8

7 How did you pay your fare? (Please cross ☒ one box only)

Adult Pass/Travelcard /Oyster/Pre-Pay ☐ 1 | Adult Cash ☐ 5
Zip/Free Under 18 yrs old ☐ 2 | Freedom Elderly /Disabled Pass ☐ 6
Saver Ticket ☐ 3 | Staff Pass ☐ 7
Child Travelcard ☐ 4

It will help us if you can answer the following questions.

8 Are you a United Kingdom resident?

Yes ☐ 1 | No ☐ 2

9 What is your home post code?

10 How many cars/vans are owned by people in your household?

None ☐ 0 | One ☐ 1 | Two or more ☐ 2

11 Are you: Male ☐ 1 | Female ☐ 2

12 How old are you?

Younger than 11 ☐ 1 | 41-50 ☐ 6
11-15 ☐ 2 | 51-60 ☐ 7
16-18 ☐ 3 | 61-70 ☐ 8
19-30 ☐ 4 | Older than 70 ☐ 9
31-40 ☐ 5

Thank you for your help.

Please hand this card in when you get off the bus
London Buses, 197 Blackfriars Road, Southwark, London SE1 8NJ

Pizza Club!

Mundher Adhami explores different approaches to division – building concepts of ‘sharing’ and ‘grouping’.

We can recognise both sharing and grouping as valid ways of reaching an answer to a division problem. However, when tackling larger numbers, children will need to develop strategies using grouping.

Concrete Preparation – Telling the story

Tell the children that when the teachers were discussing the next coffee morning for adults in school, they had an idea to organise a ‘pizza afternoon’ for children. Have a brief conversation about whether they think that would be a good idea. Talk about where it could take place within the school and clarify that tables would be needed. The pizzas will need to be organised fairly onto the tables.

Cognitive challenge – Presenting the initial problem

‘There are plenty of tables so you can use as many as you want. If we ordered 15 pizzas, how could we organise them fairly onto some tables?’

Allow children to work in pairs to discuss possible solutions and explore possible ways of recording their ideas using words, diagrams or numbers.

Share ideas – different ways of recording and explaining.

When a child simply presents the mathematical equation, ask what that means in terms of pizzas and tables. Record all versions of reasoning and ways of presenting ideas.

Mini-reflection:

Ask children – ‘Did you decide first how many tables you wanted or how many pizzas you wanted on each table? Why?’

Look at the equation together: $15 \text{ divided by } 3 = 5$

Discuss that it could represent either:

15 pizzas organised onto 3 tables giving 5 pizzas on each table

or

15 pizzas organised into 3s needing 5 tables.

‘Do you agree? Which way was easier to think about?’

Develop ideas using larger numbers:

‘What if we ordered 24 pizzas?’

Again give time for discussion and share ideas as above.

Explore patterns that emerge.

‘What if we ordered 65 pizzas?’

Again give time for discussion and share ideas as above.

Again ask ‘Did you decide first how many tables you wanted or how many pizzas you wanted on each table? Why?’

Look at methods.

Find an example of children counting in 5s to get to 65 to see how many 5s fit into 65.

Prompt children into thinking about a ‘shortcut’ (using chunking)

$10 \times 5 = 50$...now we need 15 more... and... $3 \times 5 = 15$, so 13 lots of 5 fit into 65.

Can we use this idea to find out

how to organise 135 pizzas onto tables fairly?

children will need to develop strategies using grouping.

Reflection

‘What helped you with your thinking to find solutions to these problems?’

‘What made it easier for you?’

‘Are there any new ideas you have understood today for the first time?’

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Book Review by Jane Gabb

The Spirit Level* *Why Equality is Better for Everyone

By Richard Wilkinson and Kate Pickett

Penguin Books

ISBN 978-0-141-03236-8

£9.99

This is an unusual book to find being reviewed in a mathematics education magazine as it is neither a mathematics book nor one about teaching. It does however contain a great deal of mathematics, mostly in the form of scatter diagrams. The authors use mathematics to back up their argument. Their theme also relates to *Equals*' philosophy of trying to equalise the educational opportunities for those who find learning difficult.

It is clear from the subtitle (see above) what the main message of this book is. The authors, who are professors at the University of York, present a powerful case for making societies more equal. They argue that this is better, not just for the poorest, but also for the majority. They use a number of measures to show this, and plot them against a scale of equality/inequality.

In an early chapter they point out that as societies have become more affluent there has been a long term rise in rates of anxiety, depression and other social problems. In focussing just on the 23 richest countries of the world, they compare various measures of health and social indicators.

As their measure of inequality they take the ratio of the income received by the top 20% of the population to that received by the bottom 20%. Under this measure

Singapore and the US emerge as the most unequal countries, followed by Portugal and the UK. The most equal societies are Japan and the Nordic countries.

Their scattergraphs show a consistent pattern of relationships between health and social problems and the measure of inequality. The relationship is not present when average national income is plotted against the health and social indices, showing that it is the inequality and not the level of average income that makes the difference. As an illustration, they show that life expectancy in developed countries has less to do with average income than it has to do with the level of inequality. (page 82)

They show similar relationships when looking at:

- Mental illness (including drug and alcohol addiction)
- Infant mortality
- Obesity
- Children's educational performance
- Teenage births
- Homicides
- Imprisonment rates

The power of this book is that it does not just present the evidence, it suggests why this might be happening in human and psychological terms. This allows readers to test out their own experiences of life in an unequal society with the authors' hypotheses.

If you are interested in looking into this more deeply, the authors have set up a website: www.equalitytrust.org.uk where there is an opportunity to debate the issues, download a lecture, see the evidence and find some suggestions for campaigning on this issue.

**as societies have become
more affluent there has
been a long term rise in
rates of anxiety, depression
and other social problems**