The Mathematical Association’s Contribution to ACME’s Consultation on ‘Bridging the Mathematics Gap’

0. **Preamble**: The Mathematical Association’s Response to the Study Programmes Consultation Q4 on what could be done to encourage more young people who have already achieved GCSE A*-C to study maths and what this provision would look like:

There will be a need for a variety of provision; existing GCE provision must not be diluted, in standard or availability. The recent growth in participation in mathematics at this level has been to the benefit of learners and the country; it should be permitted to continue and not be impeded.

The new post-16 learners of mathematics need new courses which relate to their needs and interests. The provision of such courses will be a major challenge to many providers and they will need support in implementation. Additionally, if learners are to be well prepared for such courses and well disposed towards them, it is important to address certain issues within the teaching of mathematics pre-16. Learners need to be brought to value further study of mathematics post-16 rather than see it as an imposition. A positive attitude to the subject needs to be developed pre-16 which values the subject and its study for its intrinsic worth (including its application) rather than as just a means of achieving examination success (with it being seen as desirable that that success be achieved as quickly as possible rather than achieving a deep knowledge to act as a firm foundation for later learning).

The new courses will need to engage the target learners; this could result in such a multiplicity of courses that many providers will not be able to afford to deliver them. A balance will need to be struck and it may be helpful if there were developed a small suite of courses addressing learners in broad subject areas but which contained elements where the learner could relate the content more closely to their other studies and interests. To this end, consideration of the structure of FSMQs, with their mix of written examination and individualised portfolios, may be fruitful.

The mechanisms for development of the new courses and qualifications need to involve all stakeholders to ensure they are mathematically and pedagogically sound, and are valued by learners, employers and further and higher education alike. (The field of post-16 education is littered with courses developed with high ideals but which never achieved wide currency; this must not happen this time.) Attention is drawn to the work that has been undertaken and is now being undertaken by ACME in this area.

Whilst the development of good courses and qualifications is a necessary condition for high-quality learning, it is not sufficient. Enough resources need to be allocated, not just for development but also for implementation. The biggest challenge will be the provision of sufficient teachers with the right skills to teach the new courses. This will require investment in large-scale programmes of initial teacher training and continuing professional development. More teachers of mathematics will be needed, lest provision is at the expense of specialist teaching for those in lower secondary years, and it will be necessary to adopt creative approaches to recruitment. The new courses are likely to need new approaches to mathematics, so even existing mathematics specialists will need to develop new skills. If implementation is to be successful, developing the teaching force must not be skimped.
1. Would you, or people you know, have benefitted from a course that bridges the post-16 maths gap?

We think that there is a need for a post-16 mathematical provision designed for students (with a grade C or above at GCSE) who do not currently take A-level mathematics. Many of these students intend to study degree courses which would benefit from greater mathematical knowledge and understanding, such as economics, social science, business, IT, life sciences and humanities. We want to know whether you agree.

- Are there qualifications that already support these students? How could these be developed, improved or supplemented to meet our needs?
- Would these students benefit from something new?
- What about other students not intending to progress to these Higher Education courses?

As there are considerable issues with students’ achievements age 5-16 (some with GCSE grades below their true capabilities, others with high grades but little real understanding), extra study for years 17-19 will not be purposeful if it is just ‘more of the same’. There is definitely a gap, and ACME’s Mathematical Needs project quantified it.

The Use of Mathematics (hereafter UoM) courses emphasise reasoning, application and communication. Currently available at AS and trialled at A2, they are composed of FSMQs which have all proved popular with students and teachers (though more challenging to teach than AS/A2). However, there seems to be an unreasonable resistance to UoM in some HE Maths quarters, because of a perceived threat to the takeup of A-Level Maths: the evidence from the Evaluating Mathematics Pathways Project (EMP) is that UoM unequivocally draws from a different cohort of students. The evidence from the Transmaths project (www.transmaths.org) is that it better supports a disposition to continue studying mathematics, and that retention is better, particularly for students with a GCSE grade B or lower. FSMQs have not had the funding support to encourage larger takeup. These level 3 UoM qualifications are suitable largely for students who already have a grade B plus at GCSE; they are not in general accessible to those with a grade C, either from Higher or Foundation tiers (and these grade Cs continue to be debased). EMP evidenced the reluctance of some students to embark on an AS qualification which did not lead to an A2, at the present time: this may change if the qualifications and university entrance regime changes significantly. The UoM might have to be repackaged to circumvent HE objections – but the evidence needs to be clearly presented in order to do so.

Level 2 UoM qualifications actually exist; they were packaged into a UoM ‘GCSE’ which was trialled very successfully but not adopted because of the regulations surrounding what can be termed a GCSE. These are very suitable for such students, who could perhaps realistically and profitably aim at higher grades.

In addition, a case could also be made for wholly new courses which would complement the arts whilst bolstering mathematical skills. Such courses might focus on the development of mathematical ideas in history which could lead to a deeper understanding of, say, the number system, and hence an improved facility with number. The history of mathematics in art and architecture might be a focus, with the geometry of perspective, for example, supporting understanding and appreciation of great paintings and buildings.
2. What mathematics do you think would prepare young people for university and the workplace? What should they be able to do by age 18? What do you wish you had learned?

Some of the reports listed in the annex at the end of this survey include suggestions for the content of a new post-16 mathematics course. We would like to know what you think such a course could contain and how it could be assessed.

- What specific topics could it be useful to include in a course designed for students not taking A-level mathematics? What should young people be able to do by the end of the course?
- What skills are required by future employers?
- Should such a course be geared towards ensuring young people are able to think with confidence in a ‘mathematical’ way, or should it focus on teaching specific mathematical techniques that are used in other subjects and vocations?
- What kinds of assessment would encourage good teaching and learning in this context?

Considerations

Both Mathematics and Use of Mathematics should be available at AS/A2, for those who want to continue with a substantial qualification in mathematics building on a GCSE grade B+, but many others, including all those studying Sciences, Psychology, Geography, Business Studies… in fact almost all Social Sciences or Engineering, should be firmly encouraged to continue with formal study of mathematics at an appropriate level for two years.

Content need not be the same for all, but inevitably smaller institutions will be able to offer less choice. Many would benefit from greater knowledge and understanding of statistics and of other modelling areas, and some from an elementary understanding of calculus, especially rates of change. Mathematics should be explored using a variety of tools, including electronic.

Possible Course Types

In Question 3 below, we define what we call for the purposes of this consultation Type 1 and Type 2 courses, reference to which should be made when reading the details in this section.

The first job of Type 1 courses is to keep students thinking mathematically. The basics need to be kept alive, in an engaging way.

Young people need more robust number and data handling skills than many have at 16, but continued development of more general mathematical ways of thinking would also benefit a substantial majority. For some, a root of apparent inadequacy is the gap between GCSE at 16 and entering employment or further education at 18; ‘use it or lose it’. For those without a substantial level 2 qualification in mathematics, a mature ‘GCSE’ based at least in part on projects, using mathematics in context, and portfolio assessment, would have credibility and be fit for purpose. This might take the form of a level 1 or 2 qualification such as the UoM ‘GCSE’ referred to above: the regulations should be driven by educational need, not vice versa, and the GCSE ‘brand’ has recognition and buy-in value.
What should be developed? These might include:

- the idea that maths can be used to model situations and solve problems; the emphasis should be on describing situations mathematically (this may require some work developing algebraic understanding) and interpreting solutions of the mathematical model (the finding of solutions would not be central to the course – the aim in this regard is to produce students who can engage intelligently and constructively with mathematical processes rather than do them)
- models relating to growth, including arithmetic and geometric (but firmly applied to real world problems and scenarios)
- probability: in the context of risk and conditionality (if they were to serve on a jury would they be able to spot either the prosecutor’s fallacy or the defender’s fallacy, what does it mean if you test positive for a disease)
- statistics: this would not be about calculation but the interpretation of presented statistics and how to detect and question critically underlying assumptions or imperfections implicit in the procedures: to be able to read and interpret intelligently and critically a statistical report.

This might be built on in Type 2 courses (which might replace GCE Use of Maths) to support students to:

- be able to apply a given model varying parameters appropriately in the model, and perhaps making minor changes to the model, and solving the model mathematically (although technical processes may well be computer-assisted)
- be able to select appropriate models from a small collection of similar models
- have some technical knowledge of statistics that would allow the student to engage intelligently in unfamiliar situations (for example, to have studied one or two examples of a hypothesis test with sufficient depth of understanding to know what issues might be relevant in selecting a hypothesis test, to be aware of the possible weakness in the underlying probability model, follow a given algorithm for the unfamiliar test and then comment intelligently on the outcome and what weight can be put on it).

Assessment

- Assessment should be developed for validity rather than reliability, with concomitant implications for selection for employment or further education. We must move to assessing what we value, if the qualifications landscape is to be better fit for purpose – but that will require significant investment in teacher CPD.
- Written timed tests are unlikely to be appropriate as the sole means of assessment. So, in addition, a portfolio of practical work, ideally validated by an oral interview conducted by a teacher from another school (in the style of the French Baccalaureate) might be suitable. Such portfolios should give students the chance to link the material studied to their other studies and interests.

Finally, in our comments in this section there are three separate points that would benefit from being drawn together. These are the advantages of using electronic tools, the importance of problem-solving through modelling and the inappropriateness of exams as the only form of assessment. Some form of portfolio/continuous assessment that involves students’ modelling to solve problems where much of the mathematical processing is outsourced to technology is needed. This would develop the skills that are desired and appear more realistic to students.
3. What ‘size and shape’ should a course for these pupils be, and how would it fit in with their other studies?

Any new course will need to fit in with, and complement, existing qualifications. We would like to understand how this could be achieved.

- Is simplicity the key or should there be a range of courses (or modules within courses) offered to students?
- What ‘size and shape’ would these courses need to be in order to fit alongside other studies (e.g. the size of an AS, but over two years)?
- How feasible is it to have a variety of course models in different types of school or college?
- How could courses be structured to enable students to transfer between the different options available at various points if they feel part-way through that another route is right for them?
- How can appropriate advice and guidance be provided?

If the overwhelming majority of students are to continue with mathematical studies post-16, then it is essential to make provision for both the spectrum of ability and the spectrum of needs and interests. Students range from those who have a substantial need for mathematics, even if not with the technical competences found in GCE Mathematics to those who have no perceptible significant need beyond those of the citizen at large.

Given the likely problems of delivery, it would seem appropriate to limit the number of course types to two. Both would be 2-year courses, packaged and presented in such a way that schools resist the strong temptation for them to be delivered in an AS slot over one session.

**Type 1** for those with moderate technical need
60-120 teaching hours over 2 years
Students without a level 2 in mathematics should be studying for the equivalent of a GCSE, in a mature form: we know that unless they can further develop their knowledge of, and confidence with, core mathematical ideas, they are likely to be to some extent marginalised in an increasingly Science-and-Technology-based world. Such courses largely exist already, though others could be developed. Students would build on their existing skills, knowledge and interests.

**Type 2** for those with greater technical need (but short of GCE Mathematics)
120-240 teaching hours over 2 years

Furthermore, we are currently locked into particular delivery patterns and structures. Perhaps others should be explored. Smaller scale study could be accommodated by changing the general expectation eg to 3 AS equivalents plus a block of ‘supporting studies’ which students could combine mathematics with, for example, a modern foreign language or General Studies. For many ordinary students, this would be far more coherent and useful than the present 4 AS subjects plus a short acquaintance with GS to get another certificate. Currently, interaction between teachers of mathematics to post-16 students and those delivering non-maths HE courses is infrequent and inadequate. Yet the value of these ‘supporting studies’ would need to be clearly stated and post-16 institutions, HE and employers would have to communicate their valuing of more mathematics. And this would need to be mirrored in funding regimes that do not necessarily value all 60-hour courses equally, for example.
4. How can young people be encouraged to study more mathematics?

Having new courses available is not enough, colleges need to offer them, and students need to choose to take them. We want to know how you would encourage more students to study mathematics.

- What practical steps should be taken to communicate the suitability of particular courses for various university subjects and other destinations?
- What steps should be taken to ensure that take-up of this course does not threaten the recent increases in numbers taking A-level Mathematics and Further Mathematics?
- Would A-level Mathematics students take these courses as well?

A significant increase in the supply of skilled specialist mathematics teachers would be necessary, and those in post would require high quality CPD too. There may be a role here for the MA, the ATM and the NCETM; and the NRICH resources which can be used to engage students would come into play.

The nesting of courses (so that Type 1 lies inside Type 2 which lies inside GCE Mathematics) may look neat to a timetabler but is likely to lead to pedagogical inefficiency, especially where those taking the more mathematically intensive courses will have to revisit material previously treated more lightly and less rigorously. Indeed it may lead to inappropriate approaches and not take advantage of opportunities that the greater technical insight could provide in understanding the broader issues. Further, having to wade through the more qualitative approaches appropriate to Type 1 (and to a lesser extent Type 2) before embarking on GCE Mathematics is likely to render that course less attractive to those who will progress to courses with a high mathematical content, and reduce the technical content that can be covered.

Funding mechanisms, and communication of valuing from end-users, are vital. It needs to be unequivocal: ‘preference is usually given to applicants who have …’, leading as soon as possible to ‘applicants are required…’. It has been done for Further Maths: there is no reason why it should not be achieved at a different level, since end-users, and the government, are clear what they want – and what we need to compete and thrive in this 21st century global economy.

Eventually, A Level Mathematics and Further Mathematics courses (and assessments) should be edited to better reflect priorities in terms of modelling and applications (but definitely not to the exclusion of pure mathematics), but that’s not a priority: these are still useable for most of the uptake. A change in assessment (and concomitant teaching methods) might well attract and retain more girls (see EMP and Transmaths evidence): in particular, while British girls with GCSE grade A (or B) are the missing students.

A Level Mathematics and UoM are aimed at different student Pathways and there is no evidence from trials that the one threatens the other, except in cases where the students concerned are manifestly unsuited to A Level Mathematics: scaremongering among the mathematics community is not helpful if a more mathematically-literate population is really desired.
5. How can we make this a reality?

We are aware that the supply of teachers could limit the numbers of young people who have access to any new courses. We want to explore the ways this could be addressed.

• What practical steps could be taken to ensure there are enough teachers to significantly increase the numbers of students studying mathematics post-16?

• Could new courses be taught by specialists in other subjects, such as geography or economics?

• Can we learn from work done by universities to prepare first year students without A-level Mathematics for their course?

We have to be prepared to compromise within the community and show a single determination. As a nation we have to invest in CPD for non-specialist teachers, and a PR job in the universities and pre-18, to ensure young people know and value the opportunities for careers within mathematics-related areas, including teaching. The teaching will be a challenge, but it is essential that we make it happen. Financial incentives should not be dismissed – the more competition we can create for mathematics-based courses and careers the better. Government rhetoric must be backed by the necessary central funding. And the mathematics community must harness all our public-profile luminaries to speak with a single voice.

Once in service, all subject teachers should be encouraged to join a subject association, receive and read their journals as one way of keeping in touch with teaching and learning developments, and be actively involved in subject association activities. Government funding in support of subject associations would likely raise standards substantially and probably out of all proportion to the sums committed.

6. In conclusion: What can you or others do to make this work? What other things do you think we should think about?

ACME’s work here is vital: it has created some significant credibility which it must work hard to maintain, and messages must be unequivocal, addressing as many practicalities as possible and getting as much of the community ‘onside’ as possible.