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Japanese Style Letter Folding

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Whileteaching English in China I received a letter from a Chinese friend folded in the unusual way shown in Fig. 1. He told me later the method was Japanese. This is how it is done:

1. Take a sheet of paper and fold it in half, but only crease it at the ends (Fig. 2).

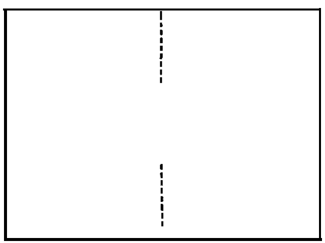

Fig. 2

2. Unfold it and then fold the opposite corners to the centre (Fig. 3).

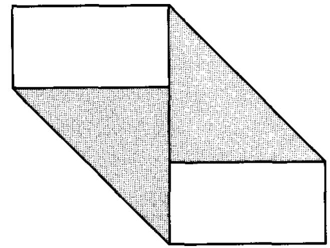

Fig. 3

3. Now fold over the top and bottom edges so that they line up with the edges of the corners you have already folded (Fig. 4).

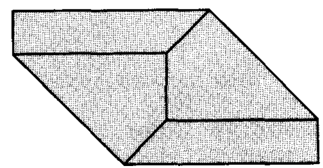

Fig. 4

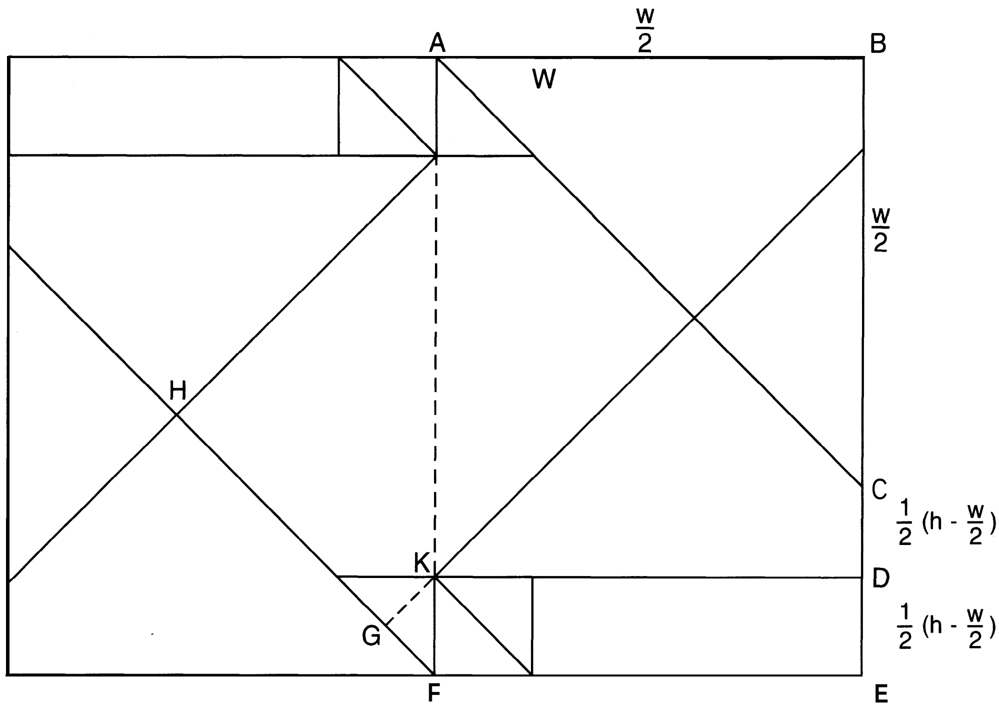

Fig. 6

4. **Then fold along the perpendicular bisector of each sloping side and tuck each tip into its pocket (Fig.** 5).

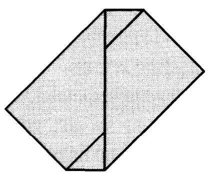

Fig. 5

You will end up with the desired result only if there are pockets for the tips to be tucked into, and for this to happen the ratio of the height of the paper to the width must be within certain limits. Finding out what these are makes an interesting investigation. One approach is to create a formula for calculating the depth of the pockets for any given values of height and width. Fig. 6 shows all the fold lines. The height of the rectangle is h, the width is w and the depth of the pockets is GK.

$$AB = BC = \frac{w}{2}$$

so $CE = h - \frac{w}{2}$ and $DE = \frac{1}{2}\left(h - \frac{w}{2}\right)$

$KF = DE$ and $GK = \frac{KF}{\sqrt{2}}$

Depth of pocket GK $= \frac{1}{2\sqrt{2}}\left(h - \frac{w}{2}\right)$

$$= \frac{2h - w}{4\sqrt{2}}$$

For this to be positive, $w < 2h$, **otherwise there would be no pockets. So the width of the paper must be less than twice the height. The minimum value of the height: width ratio must be 1 : 1, because the method will not work if** there is any gap between the opposite corners when they **are folded over, so** $h \leq w$. **If there is a gap, you cannot make the folds that produce the shape shown in Fig.** 4. **Combining these two in equations gives us** $h \leq w < 2h$.

A further exercise is to find the height and width of the folded paper in terms of h and w. Referring again to Fig. 6:

AK=BC+CD

$$= \frac{w}{2} + \frac{1}{2}\left(h - \frac{w}{2}\right)$$

$$= \frac{h}{2} + \frac{w}{4}$$

JK $= \frac{AK}{\sqrt{2}}$

$$= \frac{h}{2\sqrt{2}} + \frac{w}{4\sqrt{2}}$$

FH=JKandFG=GK

Height GH = FH – FG

$$= \frac{h}{2\sqrt{2}} + \frac{w}{4\sqrt{2}} - \frac{2h - w}{4\sqrt{2}}$$

$$= \frac{w}{2\sqrt{2}}$$

Width GJ = JK + GK

$$= \frac{h}{2\sqrt{2}} + \frac{w}{4\sqrt{2}} + \frac{2h - w}{4\sqrt{2}}$$

Substituting these values of height and width in $h \leq w < 2h$ **gives** $w \leq 2h < 2w$ **for the folded sheet, showing that now the height must be at least half the width but not more than the width itself.**

For metric paper sizes, the height : width ratio is always 1 : $\sqrt{2}$ **and if you use this paper the dimensions of the folded sheet will be in the same ratio. Both height and width will be half those of your original sheet. So, if you use a piece of A4 paper, the end result will be A6 (except for the missing corners).** ❑

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