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Pictorial integration of $\cos^2$ and $\sin^2$

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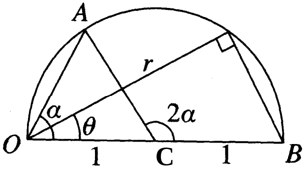
Keywords: integration

SO.54 Pictorial integration of $\cos^2\theta$ and $\sec^2\theta$

1. $\int \cos^2\theta \, d\theta$



$r = 2\cos\theta$

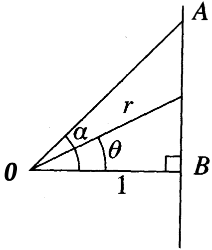
$\int_0^\alpha \cos^2\theta \, d\theta = \frac{1}{2}\int_0^\alpha \frac{1}{2}r^2 d\theta$

$= \frac{1}{2}$ (Area ***OAC*** + Area sector ***ACB***)

$= \frac{1}{2}\left(\frac{1}{2}.1^2.\ \sin 2\alpha + \frac{1}{2}.1^2.2\alpha\right)$

$= \frac{1}{2}\alpha + \frac{1}{4}\sin 2\alpha.$

2. $\int \sec^2\theta \, d\theta$



$r = \sec\theta$

$\int_0^\alpha \sec^2\theta \, d\theta = 2\int_0^\alpha \frac{1}{2}r^2 d\theta$

$= 2 \times$ Area OBA

$= 2 \times \frac{1}{2}.1.\ \tan\alpha$

$= \tan\alpha.$

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