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# Pictorial integration of $\cos^2$ and $\sin^2$

by Nick Lord



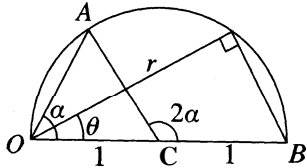
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## S0.54 Pictorial integration of $\cos^2 \theta$ and $\sec^2 \theta$

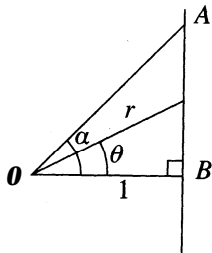
1.  $\int \cos^2 \theta d\theta$



$$r = 2 \cos \theta$$

$$\begin{aligned} \int_0^\alpha \cos^2 \theta d\theta &= \frac{1}{2} \int_0^\alpha \frac{1}{2} r^2 d\theta \\ &= \frac{1}{2} (\text{Area } \mathbf{OAC} + \text{Area sector } \mathbf{ACB}) \\ &= \frac{1}{2} \left( \frac{1}{2} \cdot 1^2 \cdot \sin 2\alpha + \frac{1}{2} \cdot 1^2 \cdot 2\alpha \right) \\ &= \frac{1}{2} \alpha + \frac{1}{4} \sin 2\alpha. \end{aligned}$$

2.  $\int \sec^2 \theta d\theta$



$$r = \sec \theta$$

$$\begin{aligned} \int_0^\alpha \sec^2 \theta d\theta &= 2 \int_0^\alpha \frac{1}{2} r^2 d\theta \\ &= 2 \times \text{Area } \mathbf{OBA} \\ &= 2 \times \frac{1}{2} \cdot 1 \cdot \tan \alpha \\ &= \tan \alpha. \end{aligned}$$